PHIL 408Q/PHPE 308D
Fairness

Eric Pacuit, University of Maryland

January 25, 2024

Course Information

Email:	epacuit@umd.edu
Website:	pacuit.org
Office:	Skinner 1103A
Office Hours:	TBA
Course Website:	https://umd.instructure.com/courses/1362256

Grading Policy

Participation	20%
Discussion	35%
Problem Sets	35%
Final Exam	10%

Grading Policy: Participation

You will be required to answer short quiz questions on PollEverywhere and ask questions and actively participate in class and/or on Piazza.

Make-up for the short quizzes will not be offered, but I will drop 5-10% of the lowest grades.

Each week, we will be discussing a different set of papers. You are responsible for learning any material covered in lectures that you miss. Please email me if you are going to miss class.

Grading Policy: Discussion

Students must submit a weekly discussion post and a response on ELMS.

Each week, you will be asked to provide a question or reaction to some aspect of the reading for this week. If you have a question, then you should give some motivation from the reading that prompted the question. Your question or reaction should be approximately 200 words.

After submitting your question or reaction, you should make at least 1 comment on another student's question or reaction.

Each discussion post is **due Wednesdays at 11:59pm**. You will. have until the following **Monday at 11:59pm** to respond to another post. I will drop the two lowest scores, so you can skip posting at most twice.

Grading Policy: Problem Set

There will be three problem sets assigned during the semester (after each part).
The problem set will consist of multiple-choice questions and short answers.

Grading Policy: Final Exam

An in-person exam given during the final exam period. A review sheet will be provided towards the end of the semester.

Required Resources

- Readings: https://umd.instructure.com/courses/1362256/modules
- PollEverywhere for in-class short quizzes. Please register at https://PollEv.com/epacuit/register?group_key=0rSZtu9dRNx0wH2TBXXTxEPkF. The registration is free.

Course Plan

1. Fairness in game theory (5 weeks)

2. Fair division (4 weeks)

3. Algorithmic fairness (6 weeks)

Course Plan

1. Fairness in game theory (5 weeks)
   How do you incorporate considerations of *fairness* into models of rational choice?

2. Fair division (4 weeks)

3. Algorithmic fairness (6 weeks)

Course Plan

1. Fairness in game theory (5 weeks)
   How do you incorporate considerations of *fairness* into models of rational choice?

2. Fair division (4 weeks)
   How do you *fairly* distribute goods to a group of people? Can we guarantee that the distribution in *envy-free*, *equitable*, and *Pareto optimal*?

3. Algorithmic fairness (6 weeks)

Course Plan

1. Fairness in game theory (5 weeks)
   How do you incorporate considerations of *fairness* into models of rational choice?

2. Fair division (4 weeks)
   How do you *fairly* distribute goods to a group of people? Can we guarantee that the distribution in *envy-free*, *equitable*, and *Pareto optimal*?

3. Algorithmic fairness (6 weeks)
   How do you ensure that algorithms make decisions without bias or discrimination?

Fairness in game theory

- Ultimatum game (and the dictator game)
- Social preferences
- Nash bargaining game
- Evolution of fairness
- Origins of unfairness

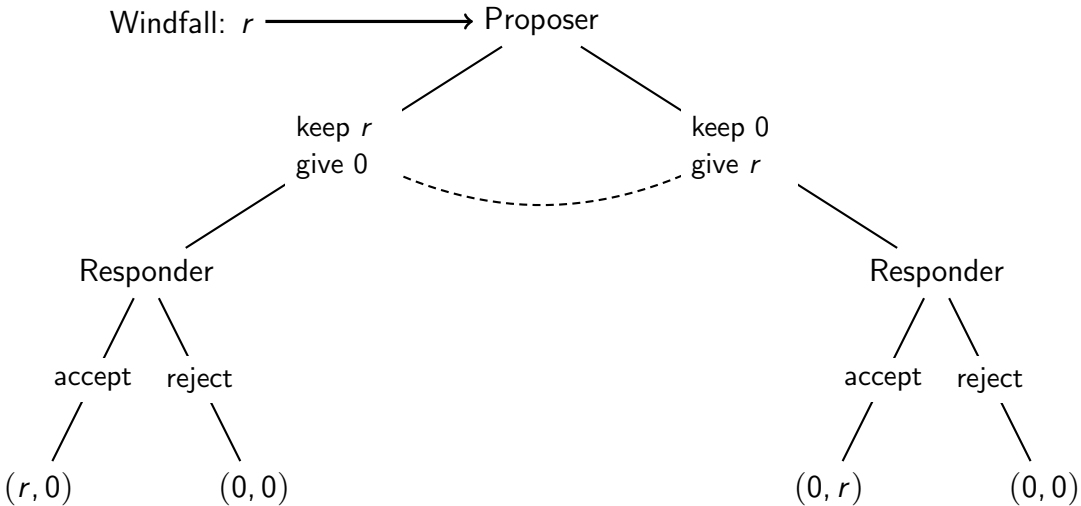
Two Important Games

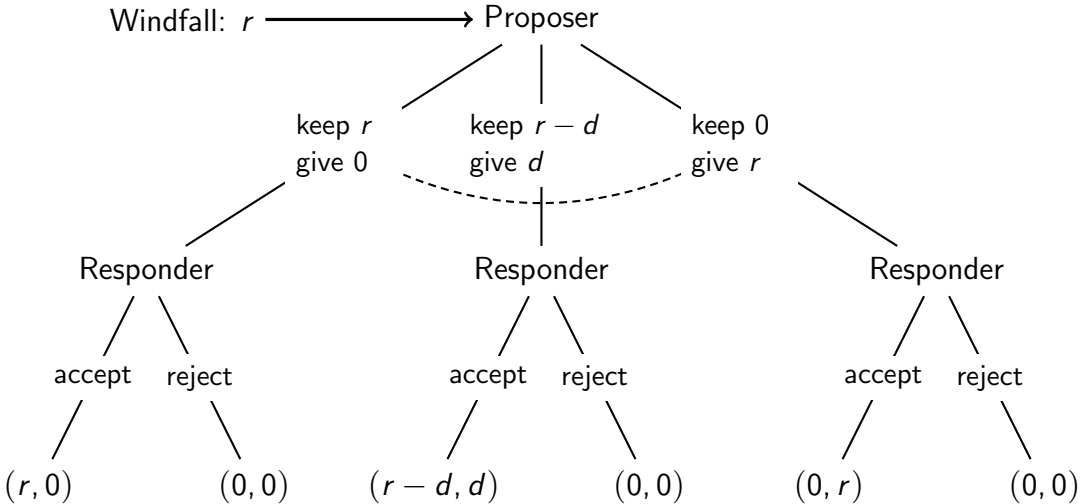
Ultimatum Game: Two players receive a windfall. One of the players suggests a division. After learning of the first player's proposal, the second must either accept or reject it. If the second accepts, both receive the amounts suggested by the first, otherwise they receive nothing.

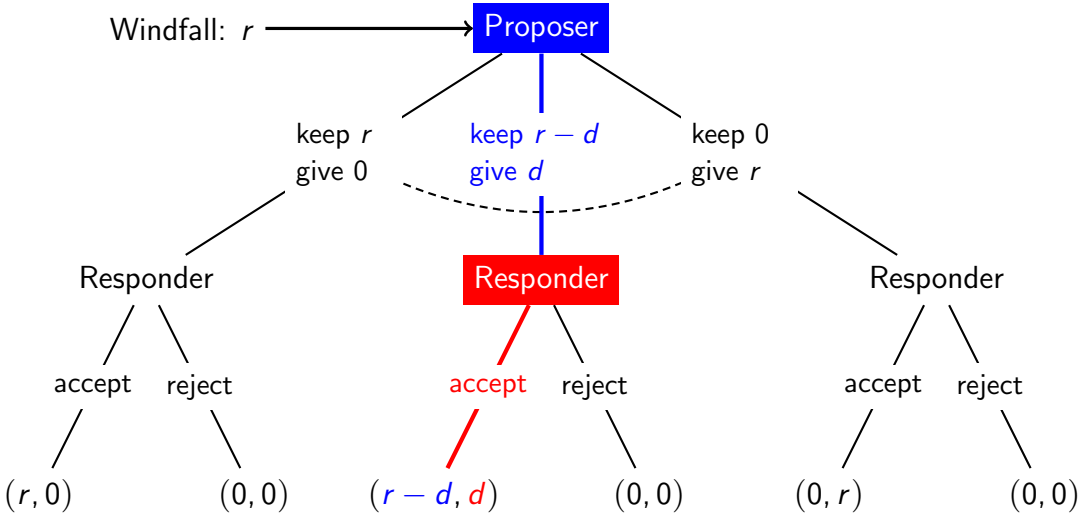
Two Important Games

Ultimatum Game: Two players receive a windfall. One of the players suggests a division. After learning of the first player's proposal, the second must either accept or reject it. If the second accepts, both receive the amounts suggested by the first, otherwise they receive nothing.

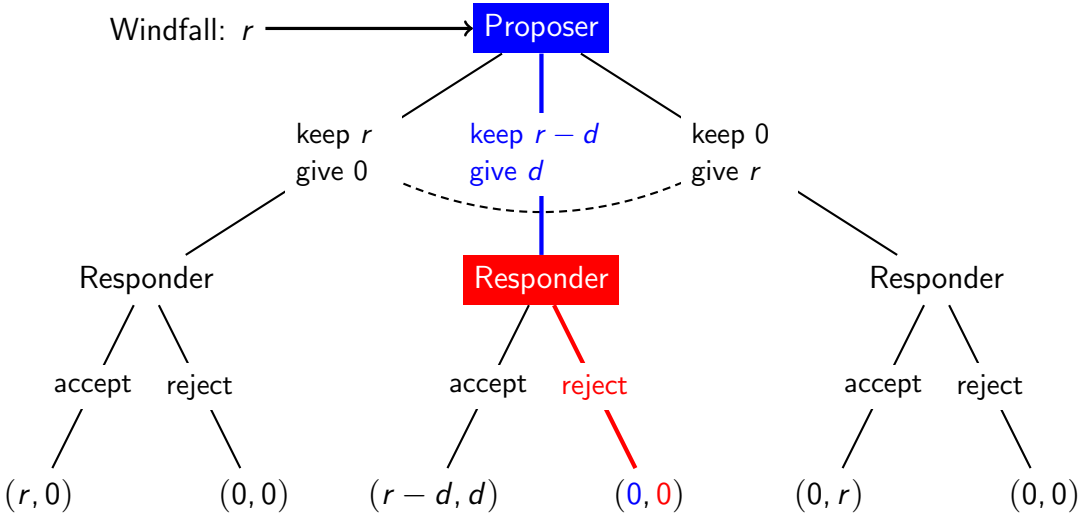
Nash Bargaining Game: Two players receive a windfall. Each player makes a demand, and if the two demands do not exceed the total good, both receive their demand. Otherwise, both receive nothing.


Windfall: r
Proposer
keep r
give 0
keep 0
give r
Responder
Responder
accept
reject
accept
reject
$(r, 0)$
$(0, 0)$
$(0, r)$
$(0, 0)$


Windfall: r
Proposer
keep r
give 0
keep $r-d$
give d
keep 0
give r
Responder
Responder
Responder
accept
reject
accept
reject
accept
reject
$(r,0)$
$(0,0)$
$(r-d,d)$
$(0,0)$
$(0,r)$
$(0,0)$



Proposer gets $r - d$ and Responder gets d



Proposer gets $r - d$ and Responder gets 0

Any strategy set where the proposer proposes to keep the most that the responder will allow is a **Nash equilibrium**.

Any strategy set where the proposer proposes to keep the most that the responder will allow is a **Nash equilibrium**.

A profile of strategies (one strategy for each player) is a **Nash equilibrium** provided that each strategy is a **best response** to the other strategies. So, no player has an incentive to deviate assuming that the other players play their strategy.

Any strategy set where the proposer proposes to keep the most that the responder will allow is a **Nash equilibrium**.

A profile of strategies (one strategy for each player) is a **Nash equilibrium** provided that each strategy is a **best response** to the other strategies. So, no player has an incentive to deviate assuming that the other players play their strategy.

The first proposer would not want to propose less, since this would result in her receiving less, nor would she want to propose more because then she would receive nothing (since, by hypothesis, the second will refuse any split that gives the first more).

Any strategy set where the proposer proposes to keep the most that the responder will allow is a **Nash equilibrium**.

A profile of strategies (one strategy for each player) is a **Nash equilibrium** provided that each strategy is a **best response** to the other strategies. So, no player has an incentive to deviate assuming that the other players play their strategy.

The first proposer would not want to propose less, since this would result in her receiving less, nor would she want to propose more because then she would receive nothing (since, by hypothesis, the second will refuse any split that gives the first more). The second player will do no better by rejecting the offer, since that would result in her receiving less.

Simplified Ultimatum Game

Three strategies for the Proposer:

1. Demand $1/3$
2. Demand $1/2$
3. Demand $2/3$

Three strategies for the Responder:

1. $[1/3, 1]$: The Responder will accept any proposal that gives her at least $1/3$
2. $[1/2, 1]$: The Responder will accept any proposal that gives her at least $1/2$
3. $[2/3, 1]$: The Responder will accept any proposal that gives her at least $2/3$

Simplified Ultimatum Game

Proposer \ Responder	$[1/3, 1]$	$[1/2, 1]$	$[2/3, 1]$
Demand $1/3$	$1/3$, $2/3$	$1/3$, $2/3$	$1/3$, $2/3$
Demand $1/2$	$1/2$, $1/2$	$1/2$, $1/2$	0, 0
Demand $2/3$	$2/3$, $1/3$	0, 0	0, 0

Simplified Ultimatum Game

Proposer \ Responder	$[1/3, 1]$	$[1/2, 1]$	$[2/3, 1]$
Demand $1/3$	$1/3$, $2/3$	$1/3$, $2/3$	$1/3$, $2/3$
Demand $1/2$	$1/2$, $1/2$	$1/2$, $1/2$	0, 0
Demand $2/3$	$2/3$, $1/3$	0, 0	0, 0

There are three Nash equilibria.

Simplified Ultimatum Game

Proposer \ Responder	$[1/3, 1]$	$[1/2, 1]$	$[2/3, 1]$
Demand $1/3$	$1/3$, $2/3$	$1/3$, $2/3$	$1/3$, $2/3$
Demand $1/2$	$1/2$, $1/2$	$1/2$, $1/2$	0, 0
Demand $2/3$	$2/3$, $1/3$	0, 0	0, 0

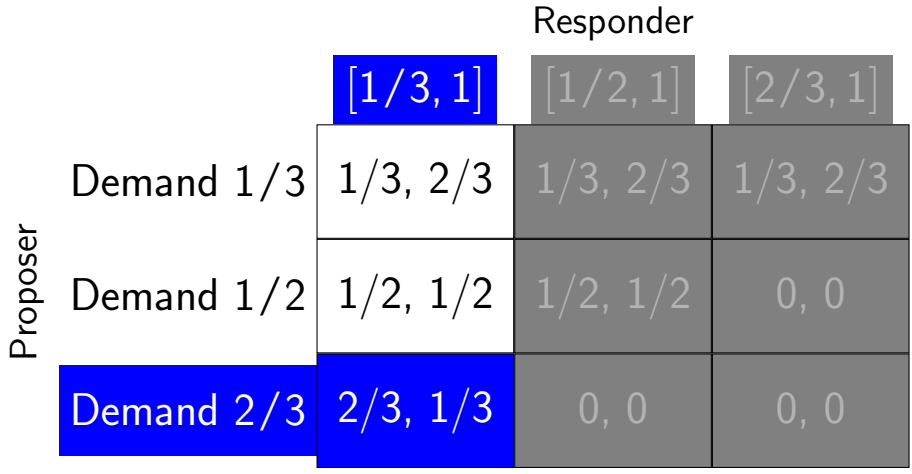
$[1/3, 1]$ **weakly dominates** both $[1/2, 1]$ and $[2/3, 1]$.

Simplified Ultimatum Game

Proposer \ Responder	$[1/3, 1]$	$[1/2, 1]$	$[2/3, 1]$
Demand $1/3$	$1/3, 2/3$	$1/3, 2/3$	$1/3, 2/3$
Demand $1/2$	$1/2, 1/2$	$1/2, 1/2$	$0, 0$
Demand $2/3$	$2/3, 1/3$	$0, 0$	$0, 0$

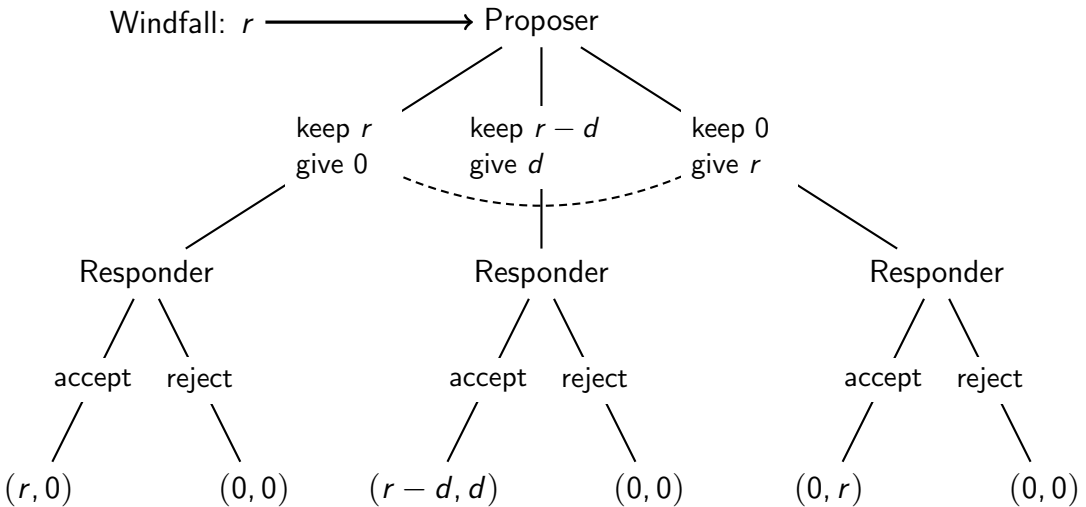
$[1/3, 1]$ **weakly dominates** both $[1/2, 1]$ and $[2/3, 1]$.

Simplified Ultimatum Game



After removing $[1/2, 1]$ and $[2/3, 1]$, Demand $2/3$ is the unique best strategy.

Sequential Rationality



Sequential Rationality

If the proposer offers a split which gives the second any positive amount, the second does strictly worse by refusing the offer. So, no rejection strategies are sequentially rational.

Knowing this, the first player ought to offer the smallest amount possible to the second player.

This is not what is observed:

> ...offers typically average about 30-40 percent of the total, with a 50-50 split often the mode. Offers of less than 20 percent are frequently rejected. These facts are not now in question. What remains controversial is how to interpret the facts and how best to incorporate what we have learned into a more descriptive version of game theory.
>
> (p. 210, Camerer and Thaler)

Hessel Oosterbeek, Randolph Sloof and Gijs van de Kuilen (2004). *Cultural Differences in Ultimatum Game Experiments: Evidence from a Meta-Analysis*. Experimental Economics 7, pp. 171–88.

Table 1. Descriptive statistics.

Variable	Obs.	Mean	Std. dev.	Min.	Max.
Year of publication	75	1998.0 *1998.4*	3.66 *3.60*	1982	2001
Year of experiment	28	1993.9 *1994.7*	3.37 *3.13*	1988	1998
Pie size in USD	57	37.12 *44.08*	86.05 *100.95*	.33	400
100 ∗ pie size in USD/GDP per capita	57	0.6527 *0.9258*	2.534 *3.296*	0.0034	17.62
Number of observations	74	31.57 *48.09*	22.99 *29.10*	3	112
Offered percentage of pie	75	40.41 *40.54*	5.85 *4.94*	26	58
Rejection rate	66	16.20 *15.75*	10.74 *10.18*	0	40
Dummy first/single round	75	0.75 *0.74*	0.44 *0.44*	0	1
Dummy strategy method	75	0.16 *0.21*	0.37 *0.41*	0	1
Dummy economics students	75	0.64 *0.65*	0.48 *0.48*	0	1

Note. In normal font are unweighted descriptive statistics; descriptive statistics in italics are weighted by number of observations of studies.

Table 1. Descriptive statistics.

Variable	Obs.	Mean	Std. dev.	Min.	Max.
Year of publication	75	1998.0 *1998.4*	3.66 *3.60*	1982	2001
Year of experiment	28	1993.9 *1994.7*	3.37 *3.13*	1988	1998
Pie size in USD	57	37.12 *44.08*	86.05 *100.95*	.33	400
100 ∗ pie size in USD/GDP per capita	57	0.6527 *0.9258*	2.534 *3.296*	0.0034	17.62
Number of observations	74	31.57 *48.09*	22.99 *29.10*	3	112
Offered percentage of pie	75	40.41 *40.54*	5.85 *4.94*	26	58
Rejection rate	66	16.20 *15.75*	10.74 *10.18*	0	40
Dummy first/single round	75	0.75 *0.74*	0.44 *0.44*	0	1
Dummy strategy method	75	0.16 *0.21*	0.37 *0.41*	0	1
Dummy economics students	75	0.64 *0.65*	0.48 *0.48*	0	1

Note. In normal font are unweighted descriptive statistics; descriptive statistics in italics are weighted by number of observations of studies.

When an alleged anomaly emerges, it is good scientific practice to test whether it is robust and not an artifact of a particular experimental environment. Based on research in the last few years, it seems fair to report that the behavior observed in the ultimatum game is quite robust. That is, different variables change the average offers and acceptances significantly, **but under no conditions are very small offers made and accepted**. (p. 210, Camerer and Thaler)

Two Important Games

Ultimatum Game: Two players receive a windfall. One of the players suggests a division. After learning of the first player's proposal, the second must either accept or reject it. If the second accepts, both receive the amounts suggested by the first, otherwise they receive nothing.

- The unique sequentially rational profile is not observed in experiments suggesting the players' decisions are influenced by considerations of fairness.

Nash Bargaining Game: Two players receive a windfall. Each player makes a demand, and if the two demands do not exceed the total good, both receive their demand. Otherwise, both receive nothing.

- Why is the 50-50 split the "obvious" solution to the Nash bargaining game?