PHIL 408Q/PHPE 308D
Fairness

Eric Pacuit, University of Maryland

March 26, 2024

Gerrymandering

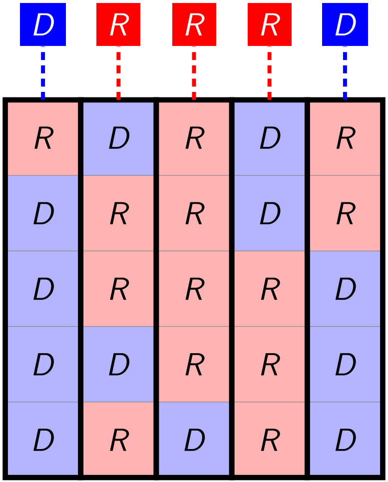
Metric Geometry and Gerrymandering Group
https://mggg.org/

The Problem

R	D	R	D	R
D	R	R	D	R
D	R	R	R	D
D	D	R	R	D
D	R	D	R	D

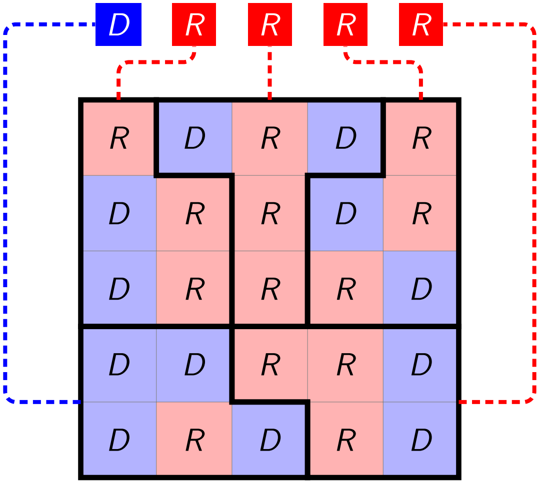
13/25 of the population will vote for R and 12/25 of the voters will vote for D.

The Problem



R wins 3 out of the 5 districts.

The Problem



R wins 4 out of the 5 districts.

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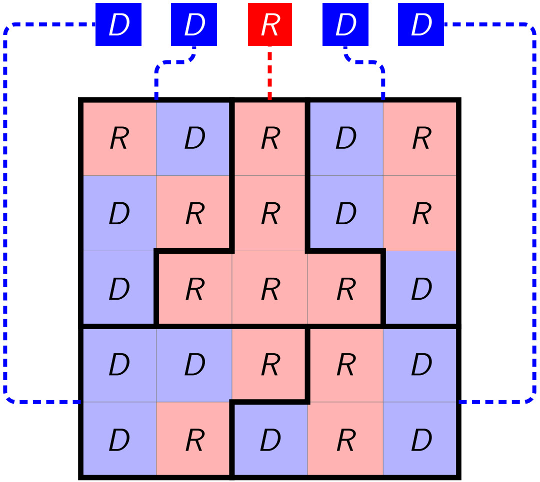

D wins 4 out of the 5 districts.

Z. Landau, O. Reid and I. Yershov (2009). *A fair division solution to the problem of redistricting*. Social Choice and Welfare, 32(3), pp. 479 - 492.

This work introduces a novel solution to the partisan unfairness problem. Instead of trying to ensure fairness by restricting the shape of the possible maps or by assigning the power to draw the map to non-biased entities, this solution ensures fairness by balancing competing interests against each other.

We stress that "fairness" of the solution presented here is not based on a fixed notion of what is desirable but rather on the preferences of the participants.

Z. Landau, O. Reid and I. Yershov (2009). *A fair division solution to the problem of redistricting*. Social Choice and Welfare, 32(3), pp. 479 - 492.

In most of the 50 states, as mentioned above, the districting protocol is to have one party draw all the boundaries; we shall call this the *single party districting protocol*.

It is this inherent unfairness of the current protocol - the ability given to the drawing party in a single party districting protocol to win a dramatically larger fractions of districts than of the constituent voters - that the districting solution proposed in this paper avoids.

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In contrast, as we shall see, the protocol proposed here ensures that either party, with knowledge of the voting map, can ensure that their party wins a percentage of districts that is very close to the percentage of support they have from the voters.

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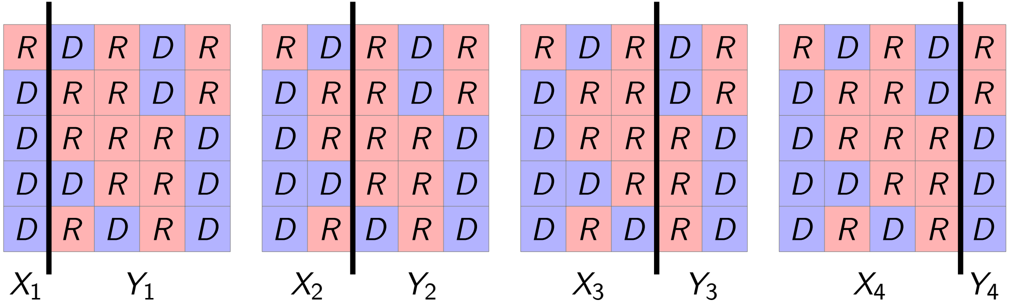
4. If no such j exists, there must be j_0 such that both players want to redistrict Y_{j_0} and X_{j_0+1}. Choose $s \in \{j_0, j_0+1\}$ at random and let a random player redistrict X_s and the other player redistrict Y_s.

Step 1



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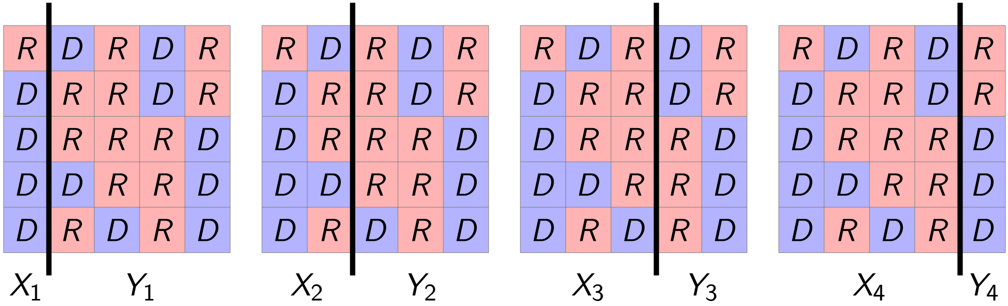
Steps 2 and 3



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Step 4

R	D	R	D	R
D	R	R	D	R
D	R	R	R	D
D	D	R	R	D
D	R	D	R	D

X_2 Y_2

R	D	R	D	R
D	R	R	D	R
D	R	R	R	D
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X_3 Y_3

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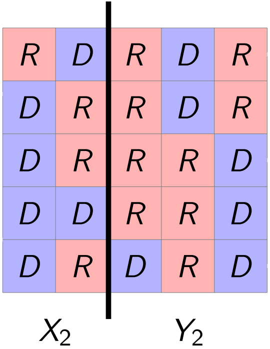
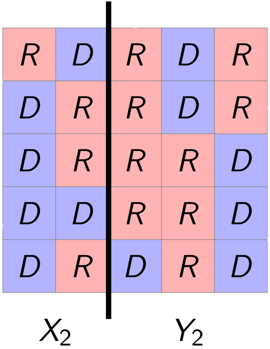


R chooses the districts in Y_2
D chooses the districts in X_2



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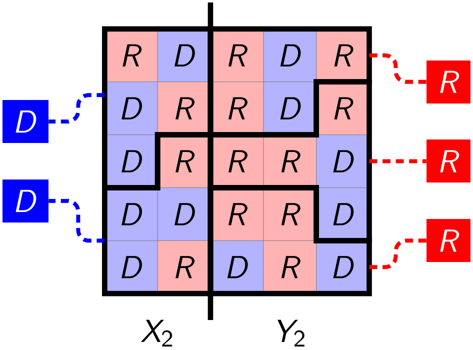

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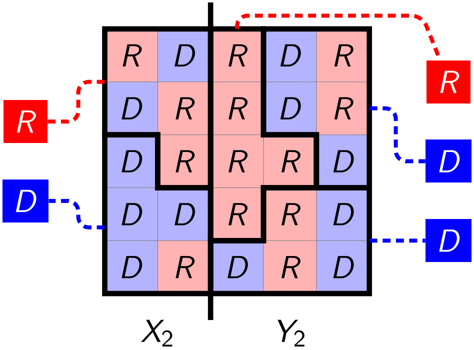

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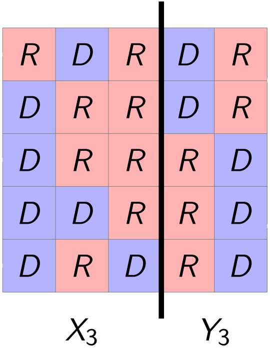
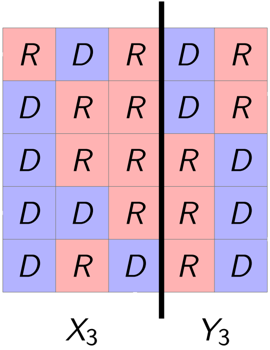


D chooses the districts in X_3
R chooses the districts in Y_3



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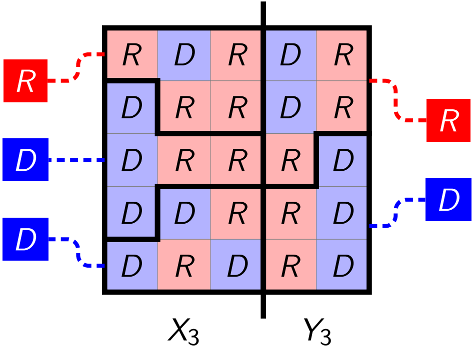

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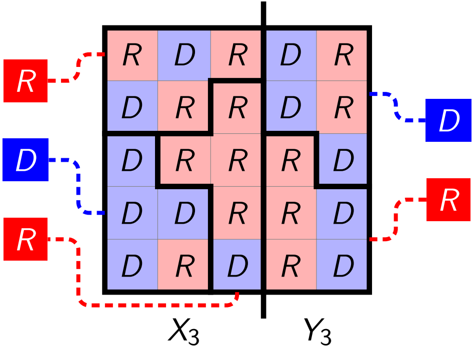

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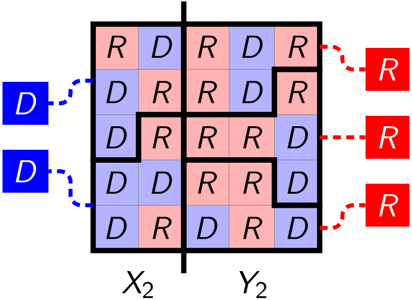
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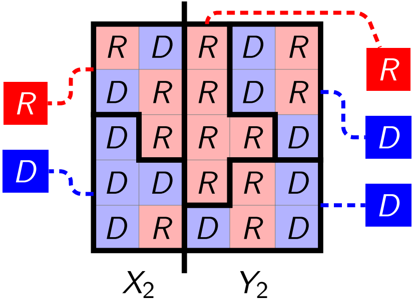
Y2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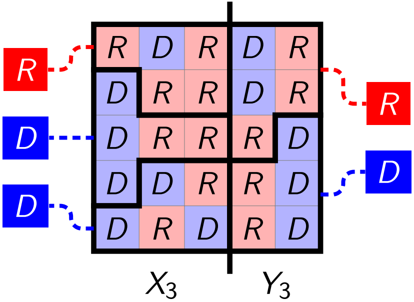
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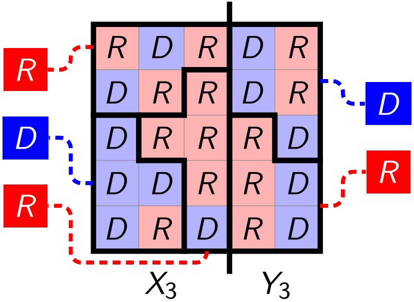
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Geometric Target

Consider a set $\mathcal{D}$ of possible partitions of a state (possibly obeying geometric constraints).

The **geometric target** of player i is the average of the maximum number of districts they can win (across all partitions in $\mathcal{D}$) and the minimum number, rounded down.

Gerdus Benadé, Ariel Procaccia, and Jamie Tucker-Foltz (2023). *You Can Have Your Cake and Redistrict It Too*. EC'23: Proceedings of the 24th ACM Conference on Economics and Computation.

Optimization Subject to Fairness

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- Possible objectives:
  - Compactness
  - Number of competitive districts
- Two obstacles:
  - How to solve the optimization problem?
  - Is the geometric target feasible in practice?

Solutions

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- does the solution have its own adverse side-effects?

Fair Representation Act

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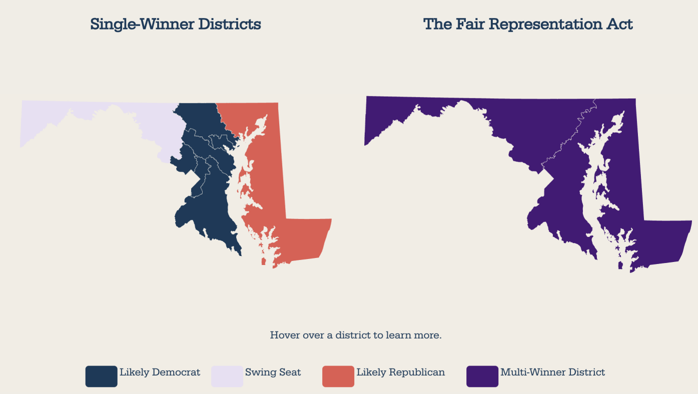
The bill requires (1) that ranked choice voting be used for all elections for Senators and Members of the House of Representatives, (2) that states entitled to six or more Representatives establish districts such that three to five Representatives are elected from each district, and (3) that states entitled to fewer than six Representatives elect all Representatives on an at-large basis.

https://fairvote.org/our-reforms/fair-representation-act/

Fair Representation Act in Maryland



https://fairvote.org/the-fair-representation-act-in-maryland/

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2. Use Single Transferable Vote (STV) to select the multiple winners giving the rankings of the voters.

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Is this a good idea?

MGGG Lab (2022). *Modeling the Fair Representation Act*. https://mggg.org/FRA-Report.

Gerdus Benade, Ruth Buck, Moon Duchin, Dara Gold, and Thomas Weighill (2021). *Ranked Choice Voting and Proportional Representation*. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3778021.

STV

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STV is not a specific voting method, but rather a family of voting methods that share certain principles:

1. Every voter should be allowed to allocate all of his vote to the candidate of his choice.
2. If a candidate has more than enough votes to be elected, then surplus votes should be transferred to the next available candidates in the rankings of those who voted for the candidate with the surplus.
3. If the candidate to whom a vote is presently allocated is excluded, then that vote should be transferred to the next available candidate in that voter's ranking.

Nicolaus Tideman and Daniel Richardson (2000). *Better voting methods through technology: The refinement-manageability trade-off in the single transferable vote*. Public Choice, 103, pp. 13 - 34.

The different STV methods vary primarily in how much of which surplus votes are transferred and in the meanings that are attached to “enough votes to be elected” and “the next available candidate”.

Example

Suppose that there are 48 voters to fill 3 positions.

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16	10	11	11
A	B	C	D
B	A	D	C
C	D	A	B
D	C	B	A

The **Hare quota** is n/k where n is the number of voters and k is the number of seats to be filled. If we use this quota, a candidate needs $48/3 = 16$ to be elected.

Then, A will be elected, *but none of the votes will be transferred to B since there is no surplus*. B will be removed, and C and D will be elected.

So the candidates elected will be $\{A, C, D\}$.

The Droop Quota

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- If two candidates are to be elected, then any candidate with more than a third of the votes ought to be assured election on the basis that there can be at most one other candidate (who can be given the other position) who will receive as many votes.

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- If two candidates are to be elected, then any candidate with more than a third of the votes ought to be assured election on the basis that there can be at most one other candidate (who can be given the other position) who will receive as many votes.
- In general, if there are k positions to be filled, then there can be at most k candidates who have more than $n/(k+1)$ votes.

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The **Droop quota** is $[n/(k+1)]+1$ where n is the number of voters and k is the number of seats to be filled. If we use this quota, a candidate needs $(48/4)+1=13$ to be elected.

Then, A will be elected, *a surplus of three votes will be transferred to B*. Then, B will be elected. One of the remaining two candidates will be selected at random.

So the candidates elected will be $\{A, B, C\}$ or $\{A, B, D\}$.

Proportionality for Solid Coalitions

Most STV methods guarantee the following property: if there is a set of voters, V, who rank all candidates in some set, S, ahead of all other candidates, then the number of candidates in S who are elected will be at least as great as the proportion of the electorate who are in V multiplied by the number of candidates to be elected, rounded down to an integer (provided that S contains at least that many candidates).

M. Dummett (1984). *Voting procedures*. Oxford University Press.