PHPE 308M/PHIL 209F
Fairness

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November 3, 2025

Problems

- Electing diverse committees
- Monotonicity
- Proportionality

Electing Diverse Committees

T. Ratliff (2006). *Selecting committees*. Public Choice, 126, pp. 242 - 255.

T. Ratliff (2003). *Some startling inconsistencies when electing committees*. Social Choice and Welfare, 21(3), pp. 433- 454.

T. Ratliff and D. Saari (2014). *Complexities of electing diverse committees*. Social Choice and Welfare, 43(1), pp. 55 - 71.

Electing Diverse Committees

Choose a committee that consists of members from different parts of the university *and* is diverse.

Social Sciences	Natural Sciences	Humanities
Ann	Carol	Ellen
Bob	David	Fred

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Ann	Carol	Ellen
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Voter 1	Voter 2	Voter 3
Ann, David, Fred	Bob, Carol, Fred	Bob, David, Ellen

Electing Diverse Committees

Choose a committee that consists of members from different parts of the university *and* is diverse.

Social Sciences	Natural Sciences	Humanities
Ann	Carol	Ellen
Bob	David	Fred

Voter 1	Voter 2	Voter 3
Ann, David, Fred	Bob, Carol, Fred	Bob, David, Ellen

Winners: Bob, David, Fred

Saari and Ratliff suggest that a way to achieve such a universally agreed upon objective is to tally ballots so that an added premium is given to each "diversity" candidate.

If each voters assigns 2 points to the "diversity" candidate and 1 point to the "non-diversity" candidates, then all committees with two males and 1 female are tied for the win.

Note that the rule makes no distinction about which category is the "diversity candidate".

Theorem (Saari and Ratliff) In electing a three-person committee from among can slotted in three divisions, suppose each division has two candidates representing two different categories (e.g., tenured and untenured faculty, or men and women, or Nordics and non-Nordics).

To reflect a universal intent shared by all voters to elect a committee with representation coming from each of these two categories, an admissible ballot must have at least one candidate from each category (so, two candidates from one category and the "diversity" candidate is from the second category).

The diversity objective always can be achieved by assigning weights if, and only if, the weights w_1 assigned to the diversity candidate equals the sum of the weights w_2 and w_3 to the other two candidates.

Increasing Committee-Size Paradox

M. Starling (1986). *Two Paradoxes of Committee Voting*. Mathematics Magazine.

Haris Aziz, Patrick Lederer, Dominik Peters, Jannik Peters, Angus Ritossa (2025). *Committee Monotonicity and Proportional Representation for Ranked Preferences*. EC'25: 26th ACM Conference on Economics and Computation, pp.896-896.

Increasing Committee-Size Paradox

1	1	1	1	1	1	1	1	1	1	1	1
a	a	a	b	b	b	c	c	d	d	e	e
f	f	g	g	h	h	h	i	i	a	b	i
c	c	c	d	d	d	e	e	e	f	g	g
h	h	h	i	f	f	f	g	g	i	i	a
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Increasing Committee-Size Paradox

1	1	1	1	1	1	1	1	1	1	1	1
a	a	a	b	b	b	c	c	d	d	e	e
f	f	g	g	h	h	h	i	i	a	b	i
c	c	c	d	d	d	e	e	e	f	g	g
h	h	h	i	f	f	f	g	g	i	i	a
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

For $k = 2$, the winning coalition according to Bloc Voting is $\{a, b\}$;

Increasing Committee-Size Paradox

1	1	1	1	1	1	1	1	1	1	1	1
a	a	a	b	b	b	c	c	d	d	e	e
f	f	g	g	h	h	h	i	i	a	b	i
c	c	c	d	d	d	e	e	e	f	g	g
h	h	h	i	f	f	f	g	g	i	i	a
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

For $k = 2$, the winning coalition according to Bloc Voting is $\{a, b\}$;
for $k = 3$, it is $\{c, d, e\}$;

Increasing Committee-Size Paradox

1	1	1	1	1	1	1	1	1	1	1	1
a	a	a	b	b	b	c	c	d	d	e	e
f	f	g	g	h	h	h	i	i	a	b	i
c	c	c	d	d	d	e	e	e	f	g	g
h	h	h	i	f	f	f	g	g	i	i	a
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

For $k = 2$, the winning coalition according to Bloc Voting is $\{a, b\}$; for $k = 3$, it is $\{c, d, e\}$; and for $k = 4$, it is $\{f, g, h, i\}$.

STV Violates Monotonicity

11	3	4	6
a	b	c	d
b	c	d	c
c	a	a	a
d	d	b	b

For $k = 1$, using the Droop quota $\lfloor \frac{24}{1+1} + 1 \rfloor = 13$, the winning winning committee is $\{c\}$.

For $k = 2$, using the Droop quota $\lfloor \frac{24}{2+1} + 1 \rfloor = 9$, the winning committee is $\{a, d\}$.

STV Violates Monotonicity

11	3	4	6
a	b	c	d
b	c	d	c
c	a	a	a
d	d	b	b

For $k = 1$, the Droop quota is $\lfloor \frac{24}{1+1} + 1 \rfloor = 13$ and the winning committee is $\{c\}$: In the first round no candidate meets the quota and so STV eliminates b. In the next round still no candidate meets the quota and STV eliminates d.

STV Violates Monotonicity

11	3	4	6
a	b	c	d
b	c	d	c
c	a	a	a
d	d	b	b

For $k = 2$, the Droop quota is $\lfloor \frac{24}{2+1} + 1 \rfloor = 9$ and the winning committee is $\{a, d\}$: In the first round STV picks a and removes it from the election together with 9 voters that rank it first. The remaining two voters that supported a transfer their votes to b, who now has plurality score 5. In the next round no candidate meets the quota and STV eliminates c. Finally, d has plurality score 10 and is selected.

Committee Monotonicity

- A lecturer lets students vote over which topics will be covered in their course. Proportionality is desirable in this context because topics should be chosen so as to keep many students interested. The lecturer expects to be able to cover 6 topics, say, but the class may proceed more quickly than expected, so the lecturer needs the flexibility to expand coverage to 7-8 topics.

Committee Monotonicity

- A multi-criteria recommender system makes recommendations based on modelling the user's preferences along several dimensions. For example, it might rank hotels based on room size, price, location, etc., with the user specifying their relative importance. The recommender system will recommend 10 hotels, say, by combining these criteria following the importance weight via proportional representation. The users can click a "show more" button that will display 10 more hotels — without thereby wanting to disqualify any of the initial 10 hotels.

Committee Monotonicity

- An award committee votes over who should receive awards. The committee has some discretion over the number of awards to hand out, depending on the strength of the nominees. Making the final decision is simplified if the voting rule is committee monotone. Proportionality is frequently desirable in these situations to cover different types of achievement.

Committee monotonicity is necessary for excellence based elections, where the goal is to choose the individually best candidates for the considered problem.

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On the other hand, “in the context of proportional representation insisting on a committee-monotone rule may prevent us from selecting a truly representative committee”:

Consider a single-peaked society (all voters and candidates can be placed along a single left-right dimension): for $k = 1$ it is most natural to select the median candidate, and for $k = 2$, the committee should intuitively consist of a “moderate left-wing” and a “moderate right-wing” candidate.

Proportionality for Solid Coalitions

The idea of this axiom is that, if there is a sufficiently large set of voters N that all prefer the candidates in a subset C to the candidates not in C, then this group should be represented by a number of candidates in C that is proportional to the size of N.

M. Dummett (1984). *Voting procedures*. Oxford University Press.

Proportionality for Solid Coalitions

A **solid coalition** for a set of candidates C is a group of voters such that candidate c' is ranked above candidate c for all voters i and candidates c' from C and candidates c not in C'.

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PSC or **Droop-PSC** A committee W satisfies **proportionality for solid coalitions** for a set of ranked ballots and a committee size k if for all integers $\ell \in \mathbb{N}$ and solid coalitions N supporting a set C with $|N| \geq \ell \cdot \frac{n}{k+1}$, it holds that $C \subseteq W$ or $|W \cap C| \geq \ell$.

Solid Coalitions

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
a	a	a	c	e	e	e	d
b	b	b	b	d	d	d	b
c	c	c	a	a	a	a	e
d	d	d	d	b	b	b	c
e	e	e	e	c	c	c	a

Solid Coalitions

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
a	a	a	c	e	e	e	d
b	b	b	b	d	d	d	b
c	c	c	a	a	a	a	e
d	d	d	d	b	b	b	c
e	e	e	e	c	c	c	a

$\{v_1, v_2, v_3\}$ is a solid coalition for $\{a\}$

Solid Coalitions

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
a	a	a	c	e	e	e	d
b	b	b	b	d	d	d	b
c	c	c	a	a	a	a	e
d	d	d	d	b	b	b	c
e	e	e	e	c	c	c	a

$\{v_1, v_2, v_3\}$ is a solid coalition for $\{a\}$

$\{v_5, v_6, v_7\}$ is a solid coalition for $\{e\}$

Solid Coalitions

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
a	a	a	c	e	e	e	d
b	b	b	b	d	d	d	b
c	c	c	a	a	a	a	e
d	d	d	d	b	b	b	c
e	e	e	e	c	c	c	a

$\{v_1, v_2, v_3\}$ is a solid coalition for $\{a\}$

$\{v_5, v_6, v_7\}$ is a solid coalition for $\{e\}$

$\{v_1, v_2, v_3, v_4\}$ is a solid coalition for $\{a, b, c\}$

Solid Coalitions

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
a	a	a	c	e	e	e	d
b	b	b	b	d	d	d	b
c	c	c	a	a	a	a	e
d	d	d	d	b	b	b	c
e	e	e	e	c	c	c	a

$\{v_1, v_2, v_3\}$ is a solid coalition for $\{a\}$

$\{v_5, v_6, v_7\}$ is a solid coalition for $\{e\}$

$\{v_1, v_2, v_3, v_4\}$ is a solid coalition for $\{a, b, c\}$

$\{v_5, v_6, v_7, v_8\}$ is **not** a solid coalition for $\{d\}$

Hare-PSC

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
a	a	a	c	e	e	e	d
b	b	b	b	d	d	d	b
c	c	c	a	a	a	a	e
d	d	d	d	b	b	b	c
e	e	e	e	c	c	c	a

$n = 8$ and suppose that $k = 2$: The Hare quota is $8/2 = 4$.

- $\{v_1, v_2, v_3, v_4\}$ is a solid coalition for $\{a, b, c\}$ and

$$|\{v_1, v_2, v_3, v_4\}| = 4 \geq 1 \times 4,$$

so at least one of these candidates must be selected.

Hare-PSC

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
a	a	a	c	e	e	e	d
b	b	b	b	d	d	d	b
c	c	c	a	a	a	a	e
d	d	d	d	b	b	b	c
e	e	e	e	c	c	c	a

$n = 8$ and suppose that $k = 2$: The Hare quota is $8/2 = 4$.

- The candidate d does not need to be included, since the solid coalition $\{v8\}$ for $\{d\}$ is too small.

Hare-PSC

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
a	a	a	c	e	e	e	d
b	b	b	b	d	d	d	b
c	c	c	a	a	a	a	e
d	d	d	d	b	b	b	c
e	e	e	e	c	c	c	a

$n = 8$ and suppose that $k = 2$: The Hare quota is $8/2 = 4$.

- $\{v_5, v_6, v_7\}$ is a solid coalition for $\{d, e\}$, but it is too small.

Droop-PSC

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
a	a	a	c	e	e	e	d
b	b	b	b	d	d	d	b
c	c	c	a	a	a	a	e
d	d	d	d	b	b	b	c
e	e	e	e	c	c	c	a

$n = 8$ and suppose that $k = 2$: The Droop quota is $\lfloor 8/3 \rfloor + 1 = 3$.

- Droop-PSC requires that the winning committee is $\{a, e\}$ as there are solid coalitions of size 3 for both $\{a\}$ and $\{e\}$.

Monotonicity and PSC

v_1	v_2	v_3
a	b	c
d	d	d
b	c	a
c	a	b

Monotonicity and PSC

v_1	v_2	v_3
a	b	c
d	d	d
b	c	a
c	a	b

For $k = 1$, PSC imposes no constraint so $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$ are all possible solutions, but many methods will select $\{d\}$.

Monotonicity and PSC

v_1	v_2	v_3
a	b	c
d	d	d
b	c	a
c	a	b

For $k = 1$, PSC imposes no constraint so $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$ are all possible solutions, but many methods will select $\{d\}$.

For $k = 3$, PSC requires selecting $\{a, b, c\}$.

Monotonicity and PSC

v_1	v_2	v_3
a	b	c
d	d	d
b	c	a
c	a	b

For $k = 1$, PSC imposes no constraint so $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$ are all possible solutions, but many methods will select $\{d\}$.

For $k = 3$, PSC requires selecting $\{a, b, c\}$.

But if you select $\{d\}$ for $k = 1$ and $\{a, b, c\}$ for $k = 3$, then you violate committee monotonicity!