

PHIL 408Q/PHPE 308D

Fairness

Eric Pacuit, University of Maryland

April 18, 2024

Brian Hedden (2021). *On statistical criteria of algorithmic fairness*. Philosophy & Public Affairs, 49(2), pp. 209 - 231.

Fairness

“I want to focus not on whether an algorithm is unfair to individuals, or whether it is unfair to groups. Rather, I want to focus on whether it is unfair to individuals *in virtue of their membership in a certain group*.”

Fairness

How does this notion of fairness differ from the others?

Fairness

How does this notion of fairness differ from the others?

- ▶ One can be unfair to an individual without being unfair to them in virtue of their group membership.

Fairness

How does this notion of fairness differ from the others?

- ▶ One can be unfair to an individual without being unfair to them in virtue of their group membership.
- ▶ It is not obvious that fairness is owed to groups, as opposed to individuals.

Fairness

How does this notion of fairness differ from the others?

- ▶ One can be unfair to an individual without being unfair to them in virtue of their group membership.
- ▶ It is not obvious that fairness is owed to groups, as opposed to individuals.
- ▶ Granting the notion of unfairness to groups, one can perhaps be unfair to an individual in virtue of their membership in a certain group without being unfair to that group itself, for instance if one treats a single individual worse because of their race or gender but at the same time takes other actions that are to the net benefit of that group.

Fairness Criterion

Some fairness criterion involve studying the internal workings of the algorithm.
E.g., the algorithm cannot be based on certain features.

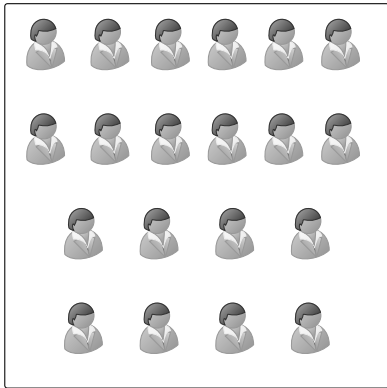
Fairness Criterion

Some fairness criterion involve studying the internal workings of the algorithm. E.g., the algorithm cannot be based on certain features.

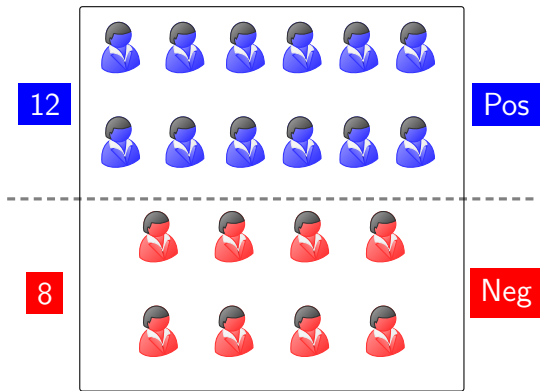
Statistical Criteria of Fairness: Criteria that require that certain relations between predictions and actuality be the same for each of the groups in question.

The criteria can be evaluated without actually looking at the inner workings of the algorithm, which may be proprietary or otherwise opaque. Instead, we just have look at the results—what the algorithm Predicted and what actually happened.

20 people

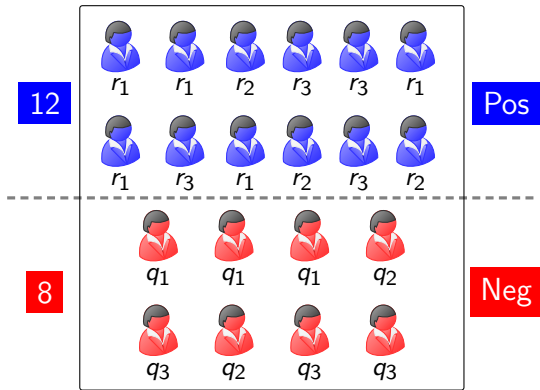


20 people



Binary predictions: 12 classified as positive (Pos); 8 classified as negative (Neg)

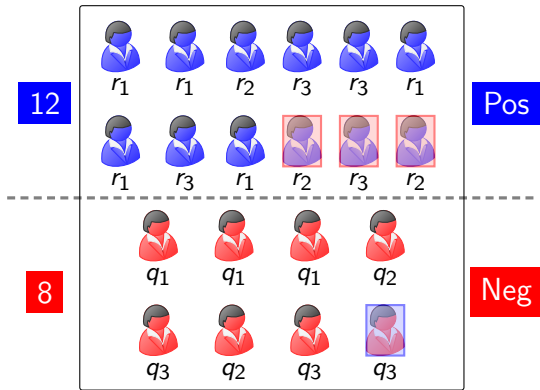
20 people



Binary predictions: 12 classified as positive (Pos); 8 classified as negative (Neg)

Predict Risk Scores: $0 \leq q_1, q_2, q_3, r_1, r_2, r_3 \leq 1$

20 people



Binary predictions: 12 classified as positive (Pos); 8 classified as negative (Neg)

Predict Risk Scores: $0 \leq q_1, q_2, q_3, r_1, r_2, r_3 \leq 1$

Actuality: 3 classified as Pos are misclassified, 1 classified as Neg is misclassified

Fairness (1)

Calibration Within Groups: For each possible risk score, the (expected) percentage of individuals assigned that risk score who are actually positive is the same for each relevant group and is equal to that risk score.

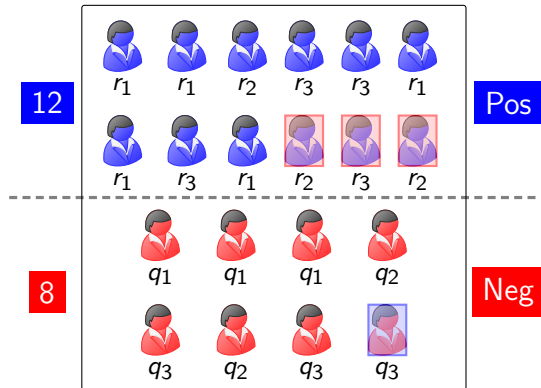
Fairness (1)

Calibration Within Groups: For each possible risk score, the (expected) percentage of individuals assigned that risk score who are actually positive is the same for each relevant group and is equal to that risk score.

The idea is that fairness requires a given risk score to “mean the same thing” for each relevant group. We want the assignment of a given risk score to have the same evidential value, regardless of the group to which the individual belongs.

Calibration

20 people



risk score	proportion Pos
r_1	1.0
r_2	1/3
r_3	3/4
q_1	0
q_2	0
q_3	1/3

Fairness (2)

Equal Positive Predictive Value: The (expected) percentage of individuals Predicted to be positive who are actually positive is the same for each relevant group.

Equal Negative Predictive Value: The (expected) percentage of individuals Predicted to be negative who are actually negative is the same for each relevant group.

Fairness (2)

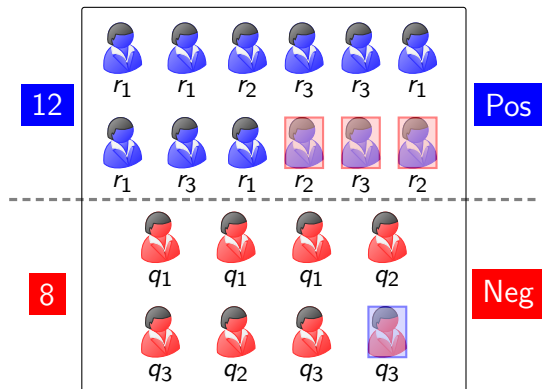
Equal Positive Predictive Value: The (expected) percentage of individuals Predicted to be positive who are actually positive is the same for each relevant group.

Equal Negative Predictive Value: The (expected) percentage of individuals Predicted to be negative who are actually negative is the same for each relevant group.

The idea is that fairness requires a prediction of positive to mean the same thing, or to have the same evidential value, regardless of the group to which the individual belongs (similarly for a prediction of negative).

Pos/Neg Predictive Value

20 people



Pos Predictive Value: $9/12$

Neg Predictive Value: $7/8$

Fairness (3)

Equal False-Positive Rates: The (expected) percentage of actually negative individuals who are falsely predicted to be positive is the same for each relevant group.

Equal False-Negative Rates: The (expected) percentage of actually positive individuals who are falsely predicted to be negative is the same for each relevant group.

Fairness (3)

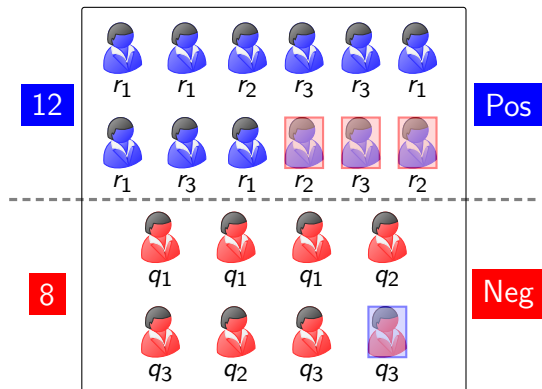
Equal False-Positive Rates: The (expected) percentage of actually negative individuals who are falsely predicted to be positive is the same for each relevant group.

Equal False-Negative Rates: The (expected) percentage of actually positive individuals who are falsely predicted to be negative is the same for each relevant group.

The idea is that fairness requires individuals from different groups who exhibit the same behavior to, on balance, be treated the same by the algorithm in terms of whether they are Predicted to be positive or negative. It would be unfair, for instance, if individuals from one group who are actually negative tended to be Predicted to be positive at higher rates than actually negative members of the other group.

False Pos/Neg Rate

20 people



False Pos Rate: $3/10$

False Neg Rate: $1/10$

Fairness (4)

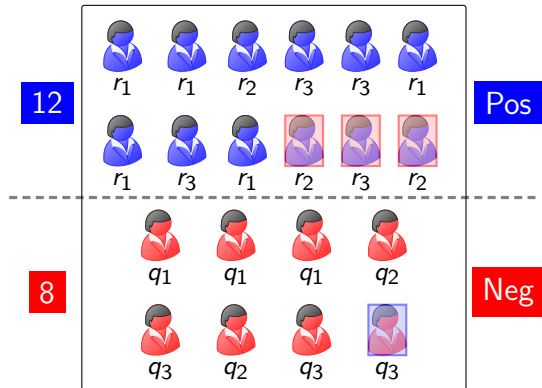
Balance for the Positive Class: The (expected) average risk score assigned to those individuals who are actually positive is the same for each relevant group.

Balance for the Negative Class: The (expected) average risk score assigned to those individuals who are actually negative is the same for each relevant group.

These are generalizations of the previous two conditions from the case of binary predictions to the case of risk scores, and are motivated in the same way.

Average Risk Scores

20 people



Average Pos Risk Score:

$$(5 * r_1 + r_2 + 3 * r_3 + q_3) / 10$$

False Neg Rate:

$$(3 * q_1 + 2 * q_2 + 2 * q_3 + 2 * r_2 + r_3) / 10$$

Fairness (5)

Equal Ratios of False-Positive Rate to False-Negative Rate: The (expected) ratio of the false-positive rate to the false-negative rate is the same for each relevant group.

Equal Overall Error Rates: The (expectation of) the number of false positives and false negatives, divided by the number of individuals, is the same for each relevant group.

Fairness (5)

Equal Ratios of False-Positive Rate to False-Negative Rate: The (expected) ratio of the false-positive rate to the false-negative rate is the same for each relevant group.

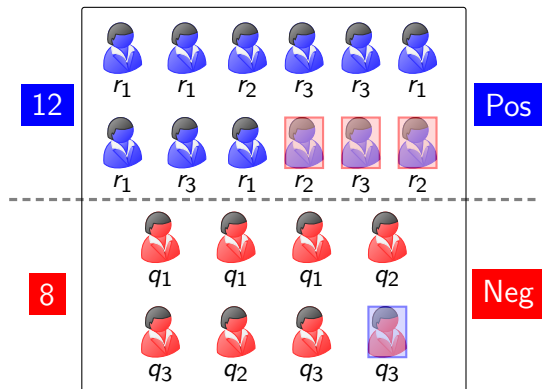
Equal Overall Error Rates: The (expectation of) the number of false positives and false negatives, divided by the number of individuals, is the same for each relevant group.

The idea is that fairness requires assigning equal relative weights to the two main error types, false positives and false negatives, for the various groups. It would be unfair, for instance, if the algorithm tended to err on the side of caution for one group while tending to do the reverse for the other group.

Equal Overall Error Rates incorporates the thought that it would be unfair if an algorithm were simply less accurate for one group than for another.

Ratio/Error Rate

20 people



False Pos to False Neg: 3 : 1

Error Rate: 4/20

Fairness (6)

Statistical Parity: The (expected) percentage of individuals Predicted to be positive is the same for each relevant group.

Fairness (6)

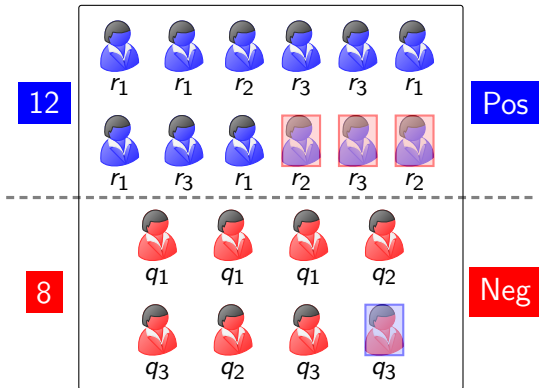
Statistical Parity: The (expected) percentage of individuals Predicted to be positive is the same for each relevant group.

The idea is that the percentage of individuals predicted to be positive be the same for each relevant group.

However, this criteria is in fact widely rejected, because it is insensitive to differences in base rates (ratios of actual positives to actual negatives) across groups. Indeed, when base rates differ across groups, this criterion will be violated by an omniscient algorithm which perfectly Predicts people's behavior. But a perfect algorithm would, presumably, not be unfair simply in virtue of differing base rates.

Percentage Predicted Positive

20 people



% Predicted Pos: 12/20

Fairness (7)

Equal Ratios of Predicted Positives to Actual Positives: The (expectation of) the number of individuals predicted to be positive, divided by the number of individuals who are actually positive, is the same for each relevant group.

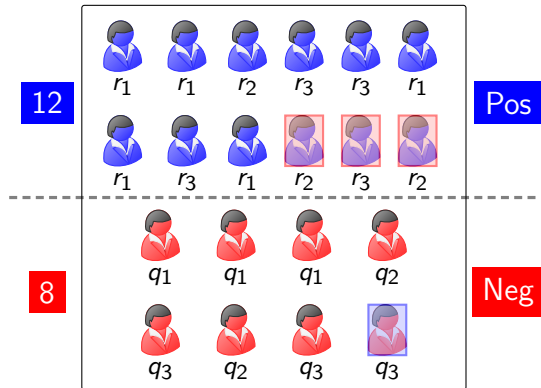
Fairness (7)

Equal Ratios of Predicted Positives to Actual Positives: The (expectation of) the number of individuals predicted to be positive, divided by the number of individuals who are actually positive, is the same for each relevant group.

This improves on the previous previous criterion. When base rates differ, this requires that differences in base rates yield corresponding differences in the rates at which individuals are Predicted to be positive.

Ratio Predicated to Actual

20 people



Predicted Pos: Actual Pos: 12/10

Impossibility

Theorem (Kleinberg, Mullainathan, and Raghavan 2016) No algorithm (for Predicting risk scores) can satisfy Calibration Within Groups, Balance for the Positive Class and Balance for the Negative Class, unless either

1. the base rates are equal across the relevant groups, or
2. the algorithm makes perfect predictions (assigning risk score 1 to all actual positives and risk score 0 to all actual negatives).

J. Kleinberg, S. Mullainathan, and M. Raghavan (2016). *Inherent trade-offs in the fair determination of risk scores*. <https://arxiv.org/abs/1609.05807>.

Impossibility

Theorem (Chouldechova 2017) No algorithm (for binary predictions) can satisfy Equal False-Positive Rates, Equal False-Negative Rates, and Equal Positive Predictive Value unless

1. the base rates are equal across the relevant groups, or
2. the algorithm makes perfect predictions (assigning 1 to all actual positives and 0 to all actual negatives).

A. Chouldechova (2017). *Fair prediction with disparate impact: A study of bias in recidivism prediction instruments*. <https://arxiv.org/abs/1610.07524>.

Impossibility

Theorem (Miconi) No algorithm can satisfy more than one of (i) Equal False-Positive Rates and Equal False-Negative Rates, (ii) Equal Positive Predictive Value and Equal Negative Predictive Value, and (iii) Equal Ratios of Predicted Positives to Actual Positives unless

1. the base rates are equal across the relevant groups, or
2. the algorithm makes perfect predictions (assigning 1 to all actual positives and 0 to all actual negatives).

T. Miconi (2017). *The impossibility of “fairness”: a generalized impossibility result for decisions.*
<https://arxiv.org/abs/1707.01195>.

“These results suggest some of the ways in which key notions of fairness are incompatible with each other.” (Kleinberg et al. 2016)

Impossibility

We might interpret these results as showing that fairness dilemmas are inevitable: whatever we do, we cannot help being unfair or biased.

Impossibility

We might interpret these results as showing that fairness dilemmas are inevitable: whatever we do, we cannot help being unfair or biased.

Alternatively, we might interpret them as showing that not all of these statistical criteria are *necessary* conditions for an algorithm to be fair or unbiased. Which criteria, then, are genuine conditions of fairness?

A Perfectly Fair Algorithm

Suppose that there are a bunch of coins of varying biases.

Each individual in the population is

1. randomly assigned a coin; and
2. randomly assigned to one of two rooms, A and B .

Goal: For each person, Predict whether that person's coin will land heads or tails. That is, our aim is to Predict, for each person, whether they are a heads person or a tails person.

Luckily, each coin comes labeled with its bias, with a real number in the interval $[0, 1]$ indicating its bias, or its objective chance of landing heads.

A Perfectly Fair Algorithm

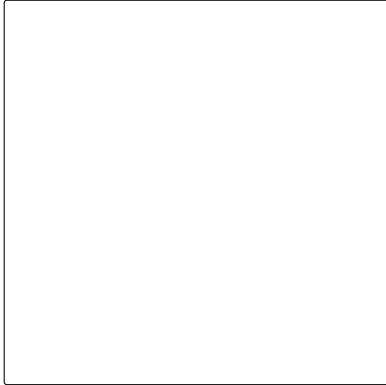
For each person, take their coin and read its label.

- ▶ If the coin label says x , assign that person a risk score of x .
- ▶ if $x > 0.5$, then Predict that they are a heads person (positive)
- ▶ if $x < 0.5$, then Predict that they are a tails person (negative).
- ▶ if $x = 0.5$, then randomize prediction (but “sidestep this issue by assuming that none of the coins are labeled “0.5”).

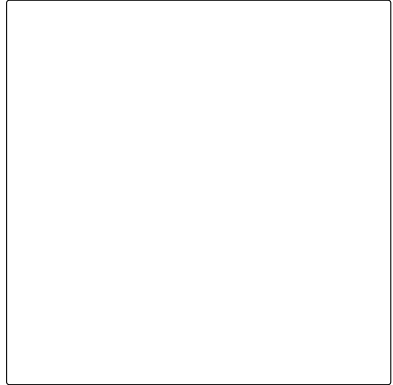
A Perfectly Fair Algorithm

- ▶ This algorithm is perfectly fair and unbiased, and in particular, it is not unfair to any people in virtue of their room membership.
- ▶ The algorithm predictions are not sensitive to individuals' room membership. And the sole feature on which its predictions are based (the labeled bias of the coin) is clearly the relevant one to focus on and is neither a proxy for, nor caused or explained by, room membership.
- ▶ Indeed, it is not just that the algorithm is in no way unfair to individuals in virtue of their membership in a certain room; there is seemingly no unfairness of any kind anywhere in this situation.
- ▶ This algorithm is uniquely optimal; no alternative can be expected to do as well or better at Predicting whether individuals are heads people or tails people.

Room A



Room B



Room A

12

     
0.75 0.75 0.75 0.75 0.75 0.75

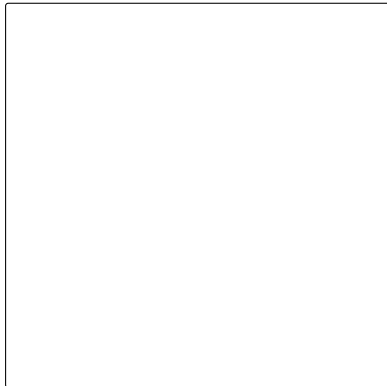
     
0.75 0.75 0.75 0.75 0.75 0.75

8

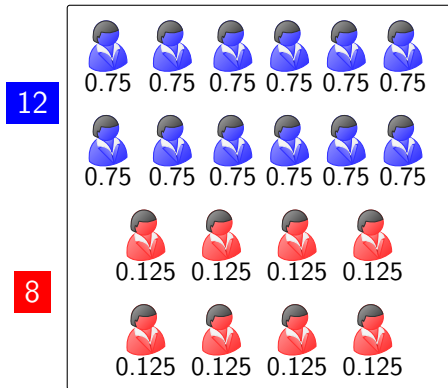
   
0.125 0.125 0.125 0.125

   
0.125 0.125 0.125 0.125

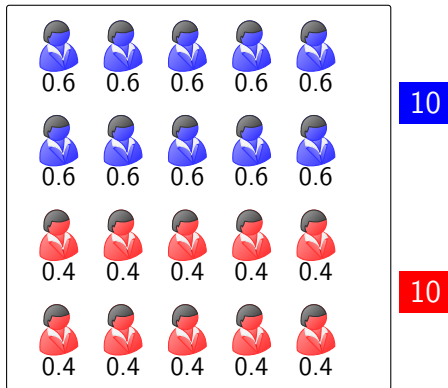
Room B



Room A



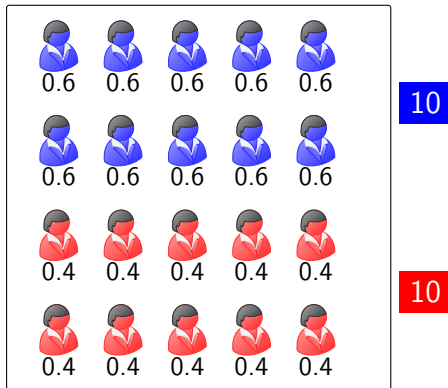
Room B



Room A

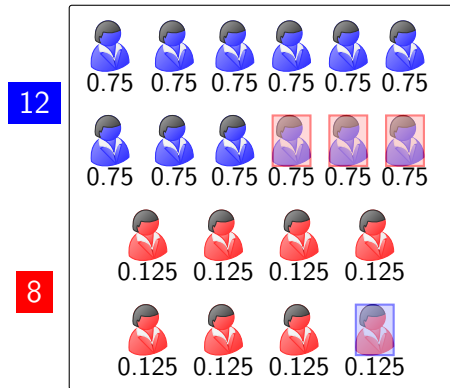


Room B



$$\begin{aligned} \text{Room A: } & 0.75 * 12 + 0.125 * 8 = 10 \text{ people are actually heads people.} \\ \text{Room A: } & 0.25 * 12 + 0.875 * 8 = 10 \text{ people are actually tails people.} \end{aligned}$$

Room A



Room B

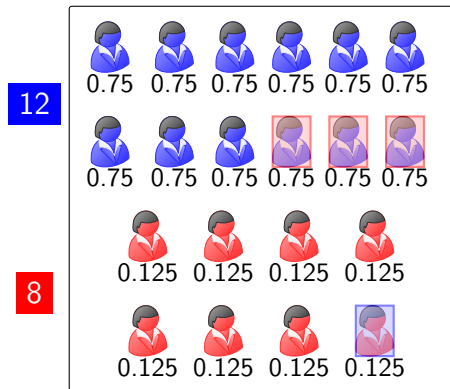


Room B: $0.6 * 10 + 0.4 * 10 = 10$ people are actually heads people.

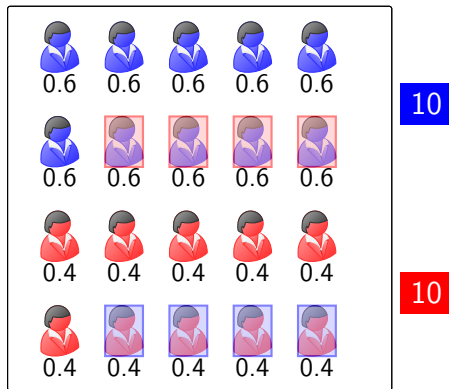
Room B: $0.4 * 10 + 0.6 * 10 = 10$ people are actually tails people.

Balance for the Positive Class is Violated

Room A



Room B



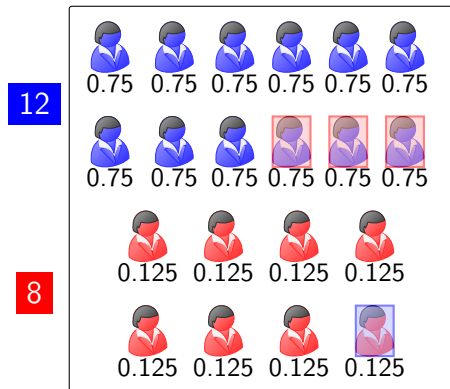
Room A

Room B

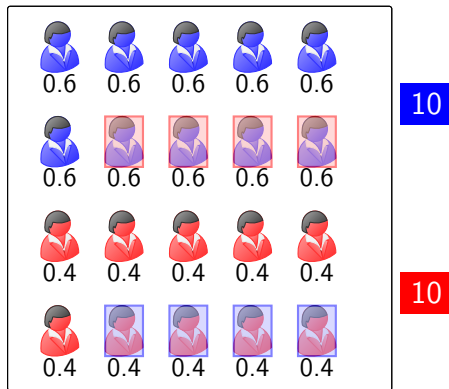
$$(9 * 0.75 + 1 * 0.125) / 10 = 0.6875 \neq 0.52 = (6 * 0.6 + 4 * 0.4) / 10$$

Balance for the Negative Class is Violated

Room A



Room B



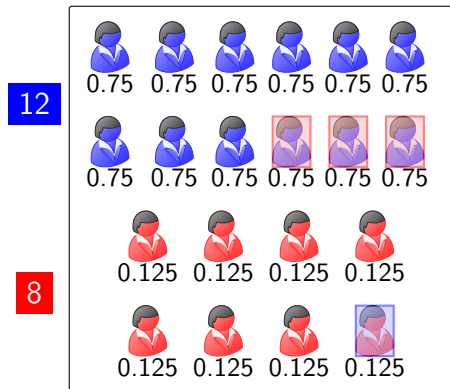
Room A

Room B

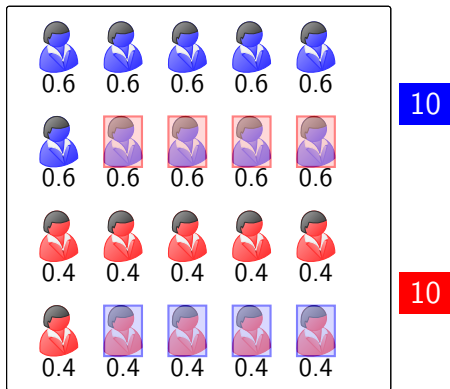
$$(3 * 0.75 + 7 * 0.125) / 10 = 0.3125 \neq 0.48 = (4 * 0.6 + 6 * 0.4) / 10$$

Equal False-Positive Rates is Violated

Room A



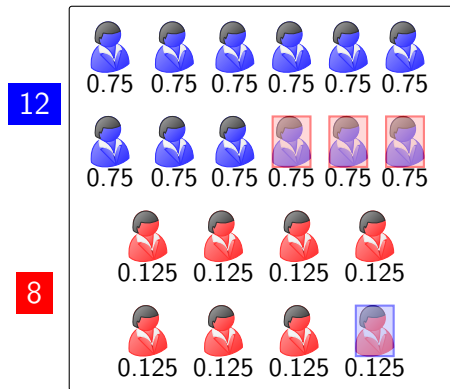
Room B



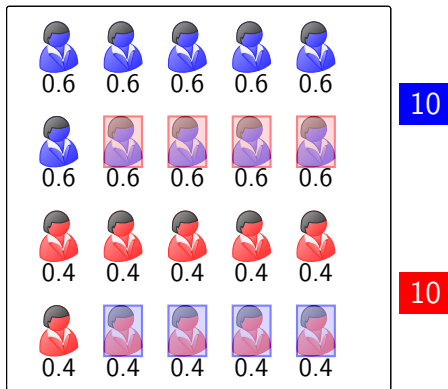
$$\frac{\text{Room A}}{\text{(False Pos Rate) } 3/10} \neq \frac{\text{Room B}}{\text{(False Pos Rate) } 4/10}$$

Equal False-Negative Rates is Violated

Room A



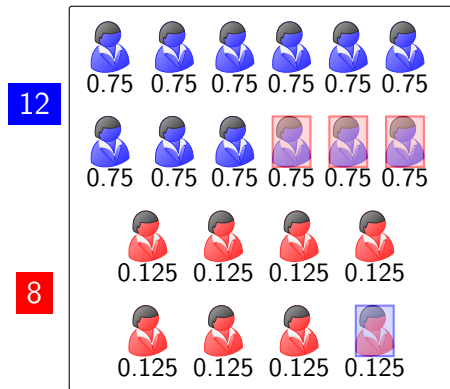
Room B



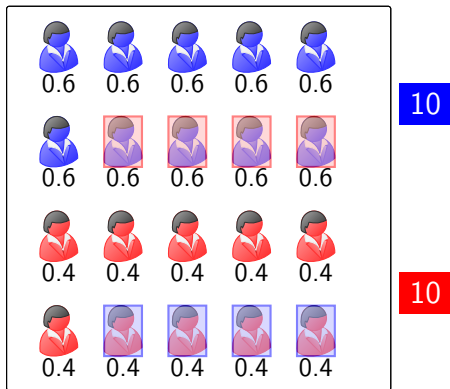
$$\frac{\text{Room A}}{(\text{False Neg Rate}) 1/10} \neq \frac{\text{Room B}}{4/10 (\text{False Neg Rate})}$$

Equal Positive Predicative Value is Violated

Room A



Room B



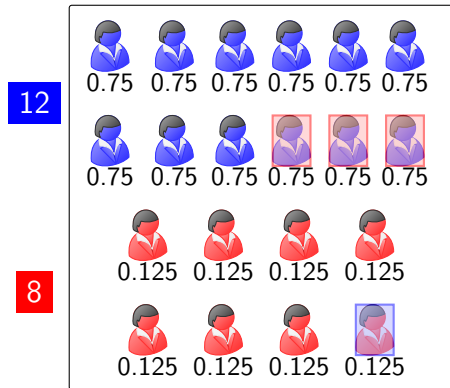
Room A

Room B

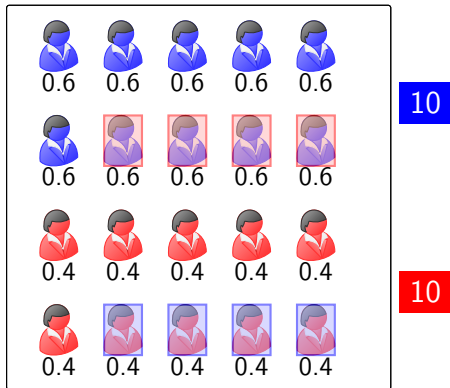
(Pos Predicative Value) $9/12 \neq 6/10$ (Pos Predicative Value)

Equal Negative Predicative Value is Violated

Room A



Room B



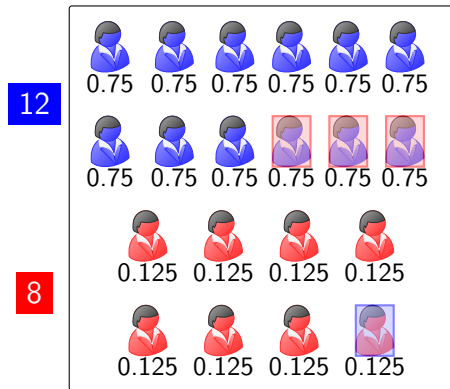
Room A

Room B

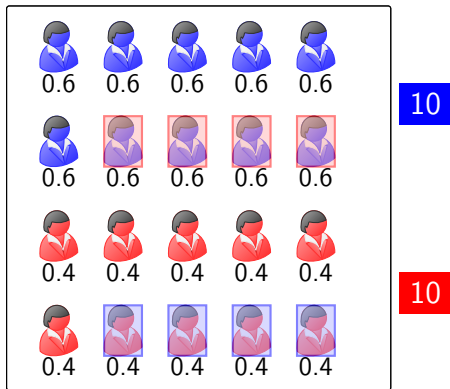
(Neg Predicative Value) $7/8 \neq 6/10$ (Neg Predicative Value)

Equal Ratios of False-Positive Rate to False-Negative is Violated

Room A



Room B



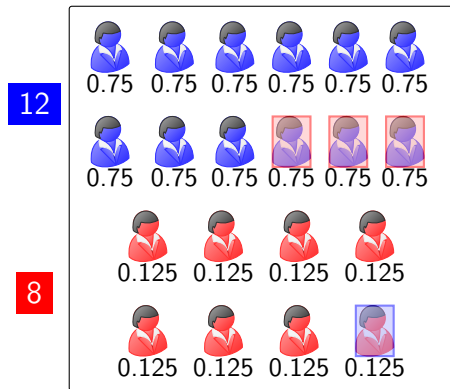
Room A

Room B

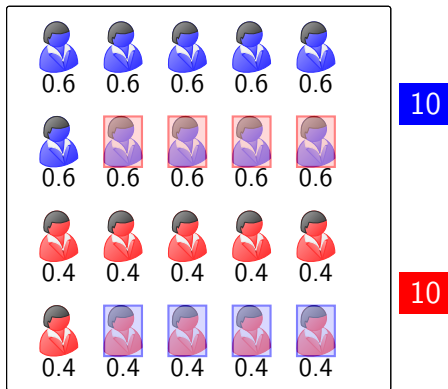
(Ratio False Pos: False Neg) 3 : 1 $\neq$ 1 : 1 (Ratio False Pos: False Neg)

Equal Overall Error Rates is Violated

Room A



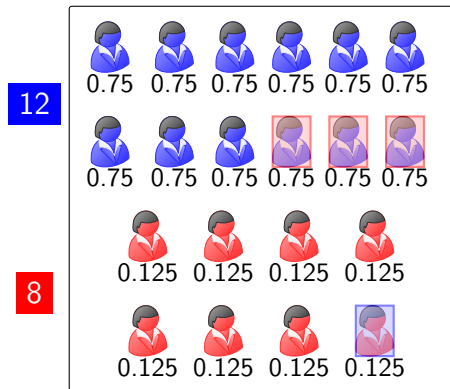
Room B



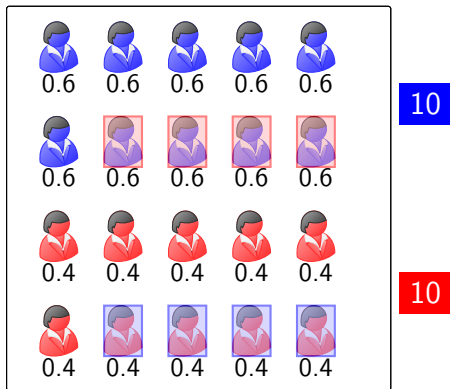
$$\frac{\text{Room A}}{\text{(Overall Error Rate) } 4/20} \neq \frac{\text{Room B}}{\text{(Overall Error Rate) } 8/20}$$

Statistical Parity is Violated

Room A



Room B



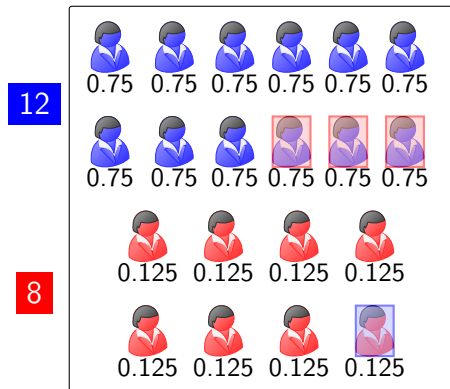
Room A

Room B

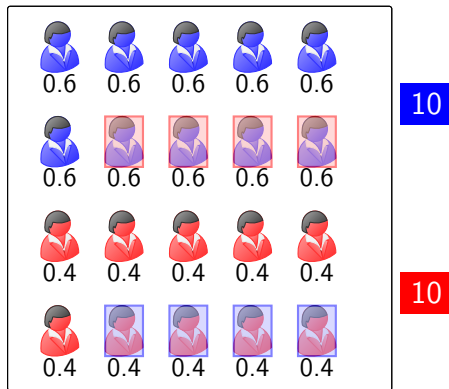
(% Predicted to be Pos) 12/20 $\neq$ 10/20 (% Predicted to be Pos)

Equal Ratios of Predicted to Actual Positives is Violated

Room A



Room B



Room A

Room B

(Ratio of Pred Pos:Actual Pos) 12 : 10 $\neq$ 10 : 10 (Ratio of Pred Pos:Actual Pos)

	Room A	Room B
Avg Score of Positives	0.6875	0.52
Avg Score of Negatives	0.3125	0.48
False Pos Rate	3/10	4/10
False Neg Rate	1/10	4/10
Pos Predictive Value	3/4	3/5
Neg Predictive Value	7/8	3/5
Ratio False Pos: False Neg	3	1
Overall Error Rate	4/20	8/20
% Predicted to be Pos	12/20	10/10
Ratio of Pred Pos:Actual Pos	12/10	10/10

It should be clear that these facts do not show that the Predicative algorithm was unfair or biased against any individuals in virtue of their being members of one room or the other.

Let me emphasize the limited nature of my argument.

I am not claiming that the case of people, coins, and rooms is realistic or completely analogous to cases like COMPAS. Of course it is not. In my example, room membership is not socially constructed, is not the basis of historical oppression, and does not influence what features people have or how they “behave” (whether their coins land heads).

But my argument does not depend on my example being realistic.

But my argument does not depend on my example being realistic.

1. simplifications and idealizations can help clarify issues by abstracting away from messy complicating factors. In real-life cases, group membership influences what features individuals have, thereby raising the thorny issue of basing predictions on “proxies” for group membership.

But my argument does not depend on my example being realistic.

1. simplifications and idealizations can help clarify issues by abstracting away from messy complicating factors. In real-life cases, group membership influences what features individuals have, thereby raising the thorny issue of basing predictions on “proxies” for group membership.
2. only arguing that none of the above criteria (except Calibration Within Groups) are necessary for fairness. And to conclude that some criterion is not necessary for fairness, all you need is a single case where fairness is satisfied but the criterion violated. That is what I have sought to provide.

Conceptual Point

When a predictive algorithm is used to make decisions with distributional consequences or other effects that we deem unfair or unjust, this does not mean that the algorithm itself is unfair or biased against individuals in virtue of their group membership.

The unfairness or bias could instead lie elsewhere: with the background conditions of society, with the way decisions are made on the basis of its predictions, and/or with various side effects of the use of that algorithm, such as the exacerbation of harmful stereotypes.

Practical Point

The best response may sometimes be not to modify the predictive algorithm itself, but to instead intervene elsewhere, by changing the background conditions of society (e.g., through reparations, criminal justice reforms, or changes in the tax code), by modifying how we act on the basis of the algorithm's predictions (e.g., by adopting different risk thresholds for different groups, above which we deny bail, or reject a loan application, and so on), or by attempting to mitigate the other negative side effects of the algorithm's use.

Suppose we face two problems: traffic and inequality.

We are deciding whether to adopt congestion pricing, which reduces traffic through extra fees for driving in the city during rush hours.

Suppose we face two problems: traffic and inequality.

We are deciding whether to adopt congestion pricing, which reduces traffic through extra fees for driving in the city during rush hours.

One might worry that this is unfair to poorer people, who may have to drive farther to work and (since they earn less) would pay a higher percentage of their incomes on congestion fees. In response, one might be tempted to abandon congestion pricing altogether, or to shift to a more complicated scheme which exempts low-income drivers.

But a better solution is available: institute the original congestion pricing scheme along with, say, a reduction in the income tax for all lower earners.

But a better solution is available: institute the original congestion pricing scheme along with, say, a reduction in the income tax for all lower earners.

We have multiple goals (reducing congestion and reducing inequality), but we also have multiple points where we can intervene. We shouldn't think that fairness demands that the congestion pricing be scrapped or that lower earners be exempted. We shouldn't ask the congestion pricing scheme to do all the work, addressing congestion and inequality at the same time.

But a better solution is available: institute the original congestion pricing scheme along with, say, a reduction in the income tax for all lower earners.

We have multiple goals (reducing congestion and reducing inequality), but we also have multiple points where we can intervene. We shouldn't think that fairness demands that the congestion pricing be scrapped or that lower earners be exempted. We shouldn't ask the congestion pricing scheme to do all the work, addressing congestion and inequality at the same time.

Of course, if it is politically or otherwise infeasible to enact this optimal policy, where congestion and inequality are addressed simultaneously but separately, it may be second best to enact the more complex congestion pricing scheme that tries to address congestion and inequality at the same time. But we should not be misled into thinking that fairness itself requires this second-best solution.

Similarly with predictive algorithms.

We have multiple aims: fair and accurate predictions, as well as just decisions and a just overall society. And we should not put excessive responsibility on the predictive algorithm itself for achieving these multiple ends.

We should, of course, ensure that the predictive algorithm achieves the first aim. But insofar as we can, we should use additional interventions elsewhere in the system to achieve the others.

Summary

The argument is that no statistical criteria, except perhaps Calibration Within Groups, is a necessary condition on fairness for predictive algorithms.

Summary

The argument is that no statistical criteria, except perhaps Calibration Within Groups, is a necessary condition on fairness for predictive algorithms.

But how we should actually design predictive algorithms depends on more than just the fairness of the algorithm itself.

In some cases, we may be able to get the results we want by just ensuring the fairness of the algorithm while making suitable interventions elsewhere.

But in other cases, we ought to design the algorithm so as to achieve certain distributional and other results.

How to go about this, however, will depend both on ethical considerations and on complex, multidimensional empirical factors not reducible to a simple formula.