PHIL 408Q/PHPE 308D
Fairness

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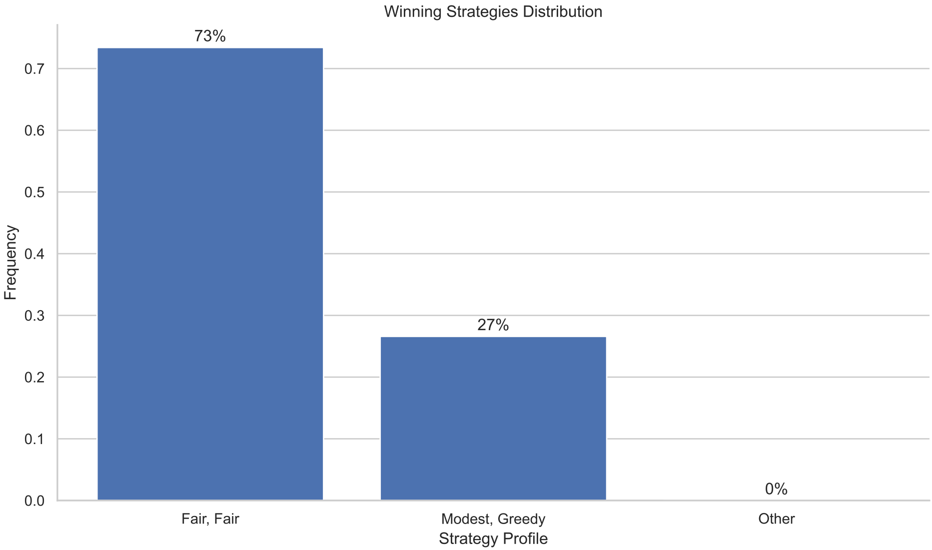
Simplified Nash Bargaining

	Modest	*Fair*	*Greedy*
Modest	$\frac{1}{3}, \frac{1}{3}$	$\frac{1}{3}, \frac{1}{2}$	$\frac{1}{3}, \frac{2}{3}$
Fair	$\frac{1}{2}, \frac{1}{3}$	$\frac{1}{2}, \frac{1}{2}$	$0, 0$
Greedy	$\frac{2}{3}, \frac{1}{3}$	$0, 0$	$0, 0$



Winning Strategies Distribution
Frequency
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
73%
27%
0%
Fair, Fair
Modest, Greedy
Other
Strategy Profile

Some Details

- $Pr_t(Modest)$: The proportion of the population playing *Modest* at time t
- $Pr_t(Fair)$: The proportion of the population playing *Fair* at time t
- $Pr_t(Greedy)$: The proportion of the population playing *Greedy* at time t

Some Details

- $Pr_t(Modest)$: The proportion of the population playing *Modest* at time t
- $Pr_t(Fair)$: The proportion of the population playing *Fair* at time t
- $Pr_t(Greedy)$: The proportion of the population playing *Greedy* at time t

- f_Modest_t: The fitness of playing *Modest* at time t
- f_Fair_t: The fitness of playing *Fair* at time t
- f_Greedy_t: The fitness of playing *Greedy* at time t

Some Details

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- $Pr_t(Fair)$: The proportion of the population playing *Fair* at time t
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- f_Modest_t: The fitness of playing *Modest* at time t
- f_Fair_t: The fitness of playing *Fair* at time t
- f_Greedy_t: The fitness of playing *Greedy* at time t

- $avg\_fitness_t$: The average fitness of the population

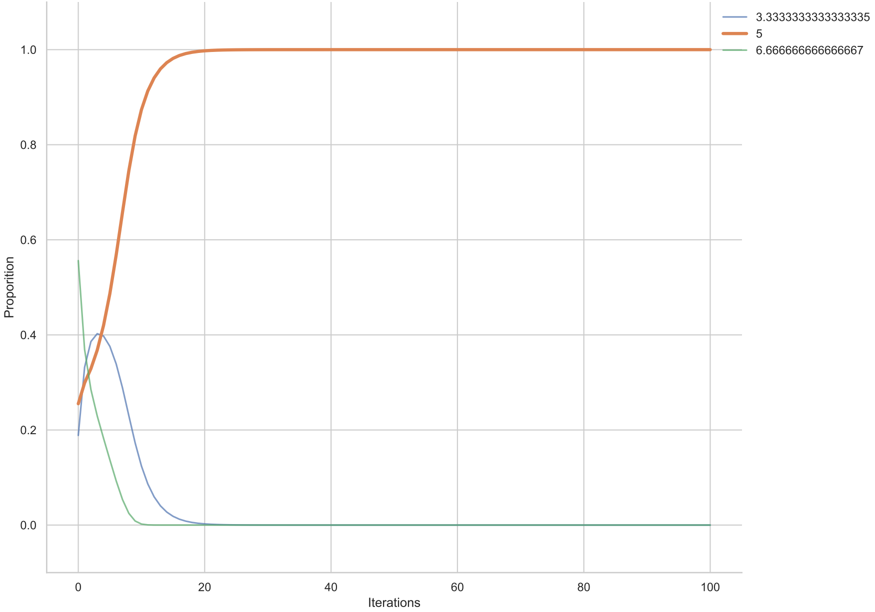
Some Details

$$
\begin{aligned}
f\_Modest_t &= Pr_t(Modest) * \frac{1}{3} + Pr_t(Fair) * \frac{1}{3} + Pr_t(Greedy) * \frac{1}{3} \\
f\_Fair_t &= Pr_t(Modest) * \frac{1}{2} + Pr_t(Fair) * \frac{1}{2} + Pr_t(Greedy) * 0 \\
f\_Greedy_t &= Pr_t(Modest) * \frac{2}{3} + Pr_t(Fair) * 0 + Pr_t(Greedy) * 0
\end{aligned}
$$

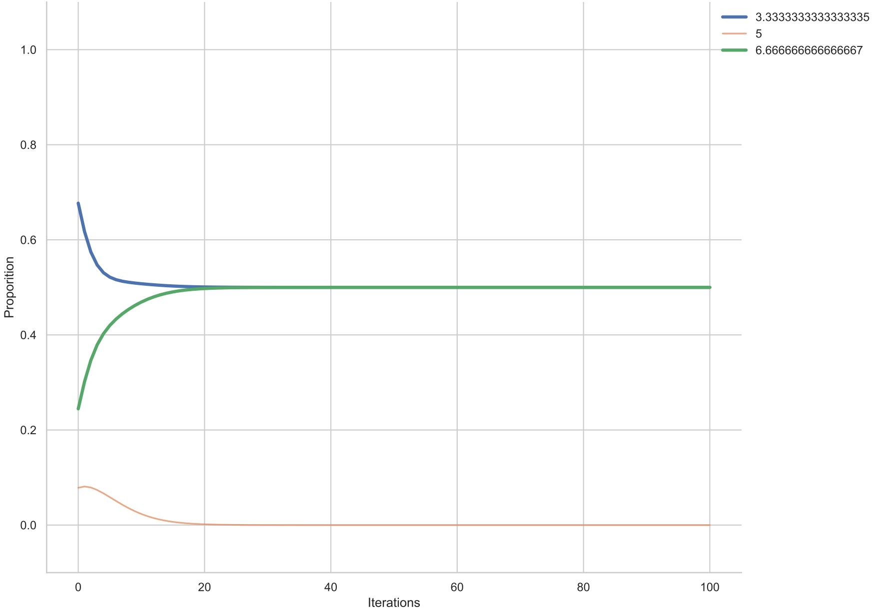
$$
\begin{aligned}
avg\_fitness_t &= Pr_t(Modest) * f\_Modest_t + Pr_t(Fair) * f\_Fair_t \\
&+ Pr(Greedy) * f\_Greedy_t
\end{aligned}
$$

Some Details

$$
\begin{aligned}
Pr_{t+1}(Modest) &= Pr_t(Modest) + Pr_t(Modest) * \frac{f\_Modest_t - avg\_fitness_t}{avg\_fitness_t} \\
Pr_{t+1}(Fair) &= Pr_t(Fair) + Pr_t(Fair) * \frac{f\_Fair_t - avg\_fitness_t}{avg\_fitness_t} \\
Pr_{t+1}(Greedy) &= Pr_t(Greedy) + Pr_t(Greedy) * \frac{f\_Greedy_t - avg\_fitness_t}{avg\_fitness_t}
\end{aligned}
$$



3.3333333333333335
5
6.666666666666667
1.0
0.8
0.6
0.4
0.2
0.0
Proportion
0
20
40
60
80
100
Iterations


3.3333333333333335
5
6.666666666666667
1.0
0.8
0.6
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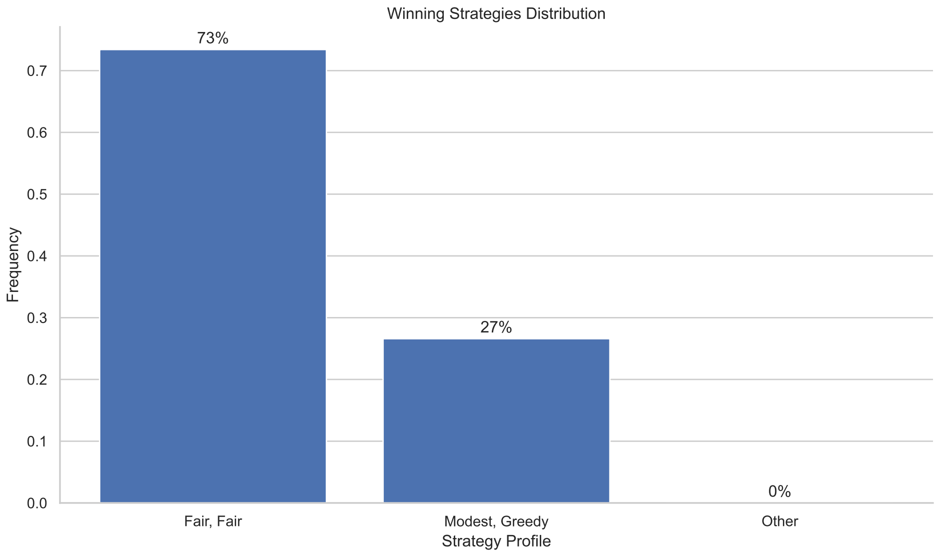
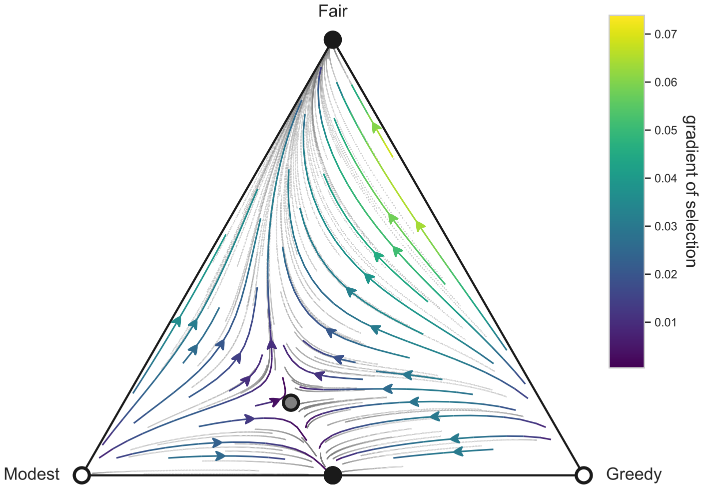
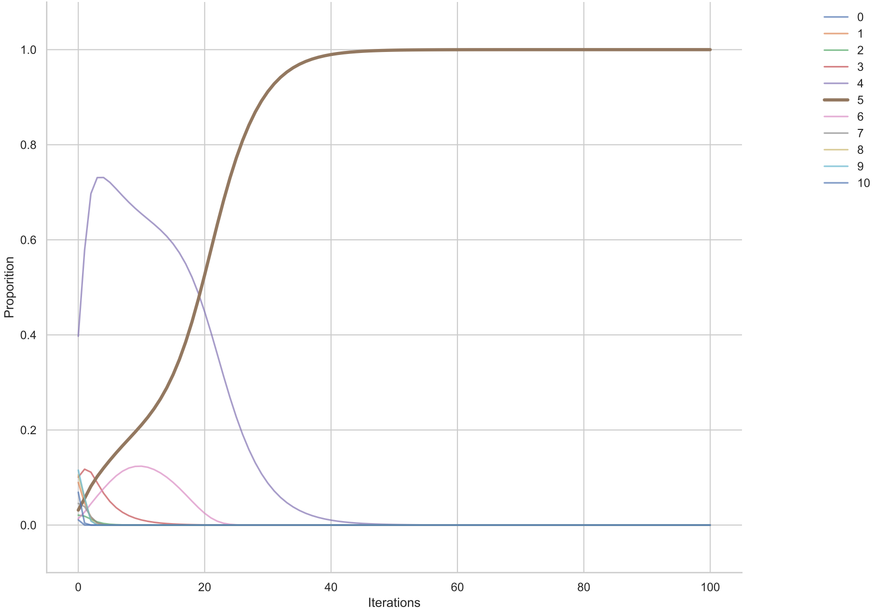
0.2
0.0
Proportion
0
20
40
60
80
100
Iterations

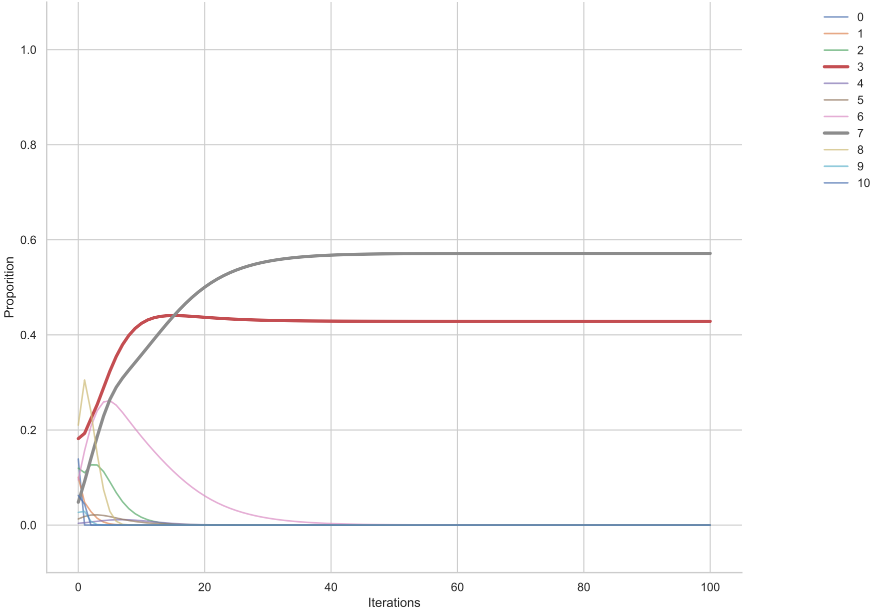

Winning Strategies Distribution
73%
27%
0%
Frequency
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Fair, Fair
Modest, Greedy
Other
Strategy Profile



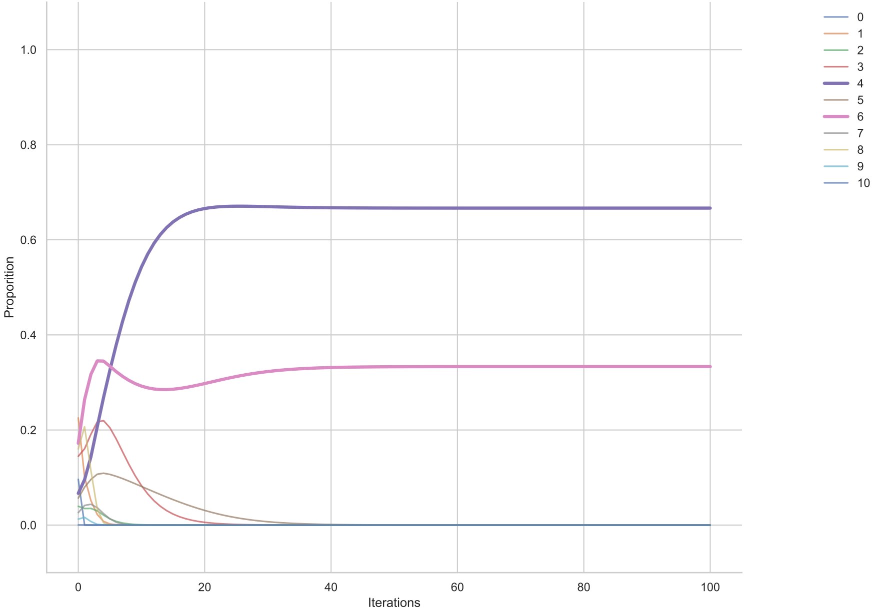
Fair
Modest
Greedy
0.07
0.06
0.05
0.04
0.03
0.02
0.01
gradient of selection

The extent of the problem of polymorphic traps depends on the granularity of a discrete bargaining game. The more slices of cake available for division, the greater the number of initial populations that will evolve to something near to fair division. If we deal with bargaining situations that are sufficiently fine grained, the problem of polymorphic traps dwindles away. (Skyrms, p. 16)

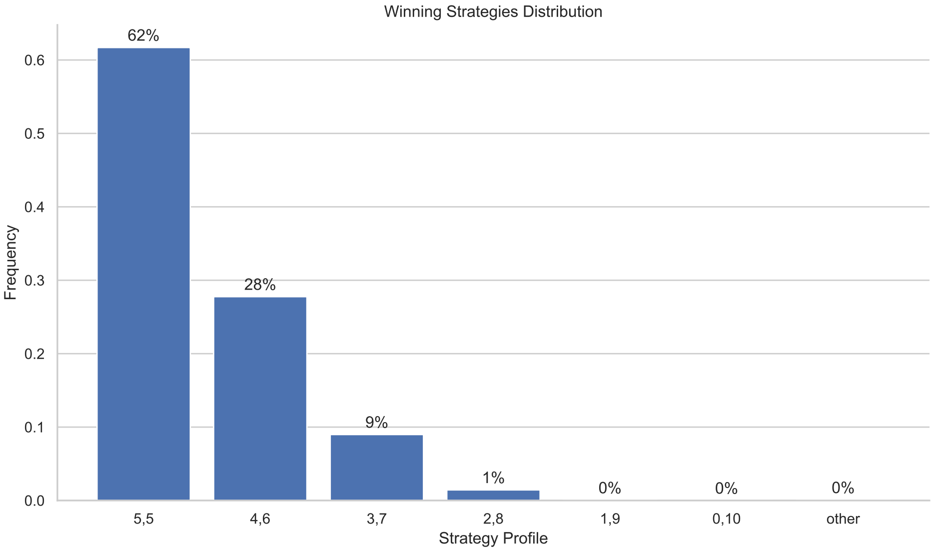

0
1
2
3
4
5
6
7
8
9
10
1.0
0.8
0.6
0.4
0.2
0.0
Proportion
0
20
40
60
80
100
Iterations

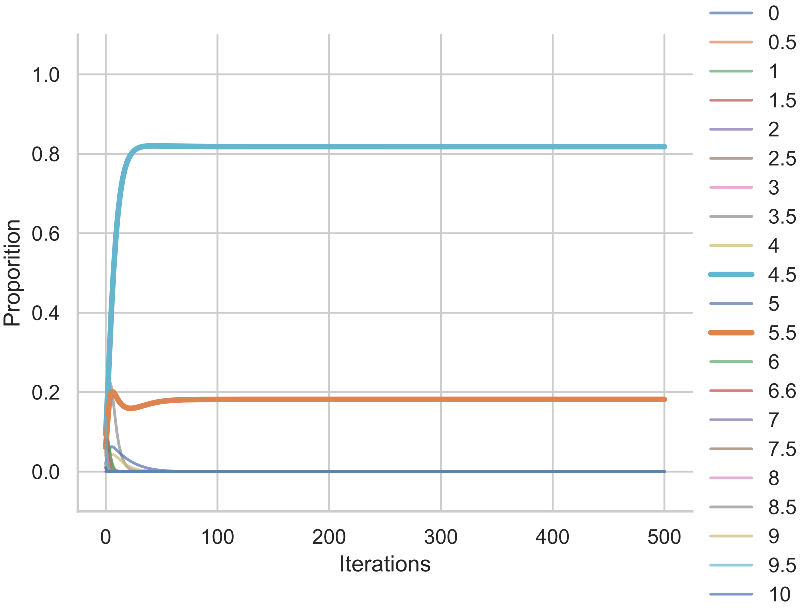


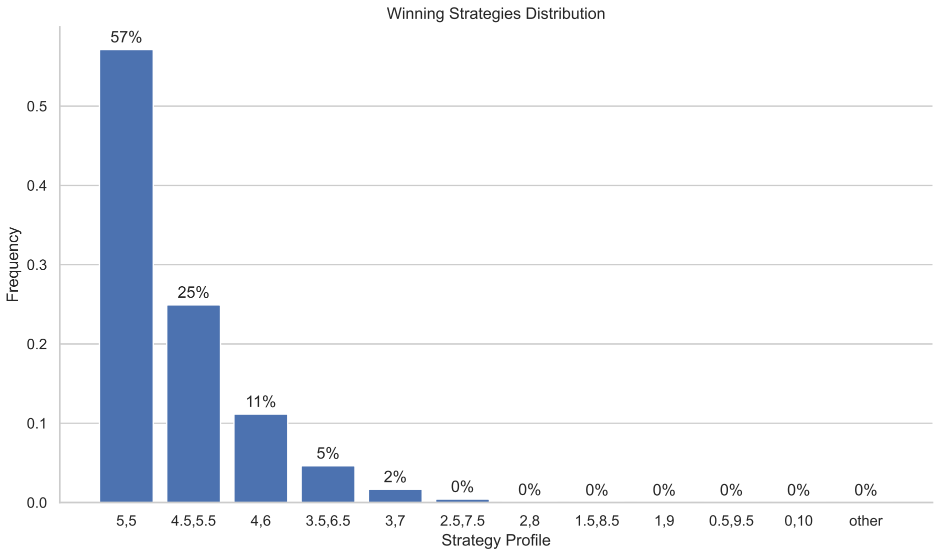
Proportion
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100
Iterations
0
1
2
3
4
5
6
7
8
9
10



Proportion
Iterations
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100
0
1
2
3
4
5
6
7
8
9
10


Winning Strategies Distribution
62%
28%
9%
1%
0%
0%
0%
0.0
0.1
0.2
0.3
0.4
0.5
0.6
Frequency
5,5
4,6
3,7
2,8
1,9
0,10
other
Strategy Profile


1.0
0.8
0.6
0.4
0.2
0.0
Proportion
0
100
200
300
400
500
Iterations
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.6
7
7.5
8
8.5
9
9.5
10


Winning Strategies Distribution
57%
25%
11%
5%
2%
0%
0%
0%
0%
0%
0%
0%
0.5
0.4
0.3
0.2
0.1
0.0
Frequency
5,5
4.5,5.5
4,6
3.5,6.5
3,7
2.5,7.5
2,8
1.5,8.5
1,9
0.5,9.5
0,10
other
Strategy Profile

Many roads lead to the egalitarian norm.

Many roads lead to the egalitarian norm. In a finite population, in a finite time, **where there is some random element in evolution**, some reasonable amount of divisibility of the good and **some correlation**, we can say that it is likely that something close to share and share alike should evolve in dividing-the-cake situations.

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Correlated Strategies

Let us...replace the assumption of random encounters with one of positive correlation between like strategies.

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It is evident that in the extreme case of perfect correlation, stable polymorphisms are no longer possible. Strategies that demand more than 1/2 meet each other and get nothing. Strategies that demand less than 1/2 meet each other and get what they demand. The fittest strategy is that which demands exactly 1/2 of the cake....

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In the real world, both random meeting and perfect correlation are likely to be unrealistic assumptions. The real cases of interest lie in between.

(Skyrms, p. 18)

Example: Correlating Strategies

Let $0 \leq \epsilon \leq 1$ be a **level of correlation**.

- $Pr_t(Modest \mid Modest)$ is the proportion playing *Modest* against *Modest*
  $Pr_t(Modest \mid Modest) = Pr_t(Modest) + \epsilon * (1 - Pr_t(Modest))$

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- $Pr_t(Fair \mid Modest)$ is the proportion playing *Fair* against *Modest*
  $Pr_t(Fair \mid Modest) = Pr_t(Fair) - \epsilon * (Pr_t(Fair))$

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- $Pr_t(Fair \mid Modest)$ is the proportion playing *Fair* against *Modest*
  $Pr_t(Fair \mid Modest) = Pr_t(Fair) - \epsilon * (Pr_t(Fair))$
- $Pr_t(Greedy \mid Modest)$ is the proportion playing *Greedy* against *Modest*
  $Pr_t(Greedy \mid Modest) = Pr_t(Greedy) - \epsilon * (Pr_t(Greedy))$

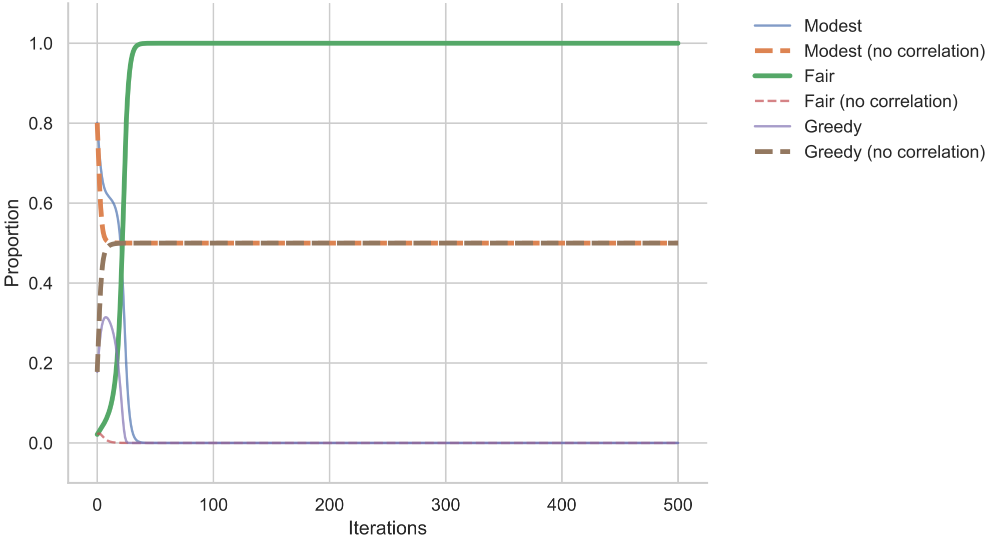
Example: Correlating Strategies

$$f\_Modest_t = Pr_t(Modest) * \frac{1}{3} + Pr_t(Fair) * \frac{1}{3} + Pr_t(Greedy) * \frac{1}{3}$$

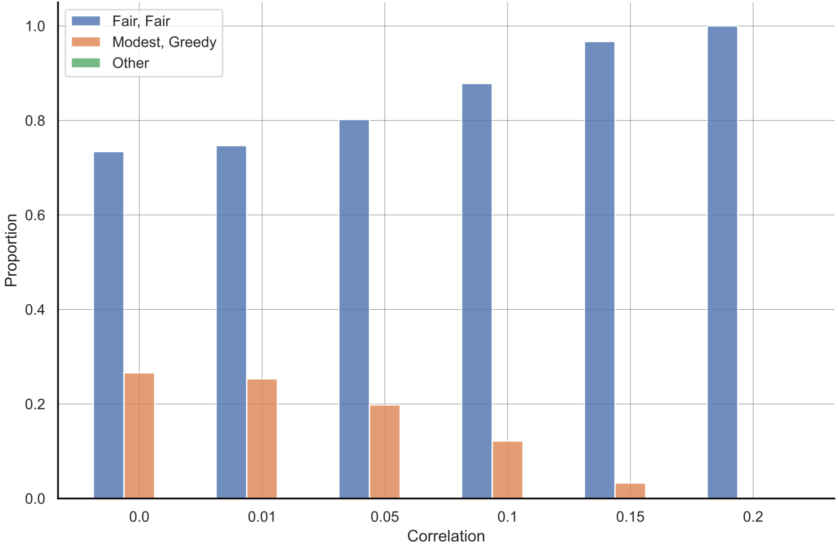
Example: Correlating Strategies

$$f\_Modest_t = Pr_t(Modest) * \frac{1}{3} + Pr_t(Fair) * \frac{1}{3} + Pr_t(Greedy) * \frac{1}{3}$$

$$Pr_t(Modest \mid Modest) * \frac{1}{3} + Pr_t(Fair \mid Modest) * \frac{1}{3} + Pr_t(Greedy \mid Modest) * \frac{1}{3}$$



Proportion
1.0
0.8
0.6
0.4
0.2
0.0
0
100
200
300
400
500
Iterations
Modest
Modest (no correlation)
Fair
Fair (no correlation)
Greedy
Greedy (no correlation)


Fair, Fair
Modest, Greedy
Other
1.0
0.8
0.6
0.4
0.2
0.0
Proportion
0.0
0.01
0.05
0.1
0.15
0.2
Correlation

Local Interaction

J. McKenzie Alexander and B. Skyrms (1999). *Bargaining with Neighbors: Is Justice Contagious*. Journal of Philosophy, 96(11).

Skyrms on the Ultimatum Game

Richard Thaler chose the ultimatum game as the subject for the initial article in a series on anomalies in economics—an anomaly being "an empirical result which requires implausible assumptions to explain within the rational choice paradigm."

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Appeal to norms of fairness, however, hardly constitutes an explanation in itself. Why do we have such norms? Where do they come from? How could they evolve?

(Skryms, p. 29)

Ultimatum Game

Proposer \ Responder	[1/3, 1]	[1/2, 1]	[2/3, 1]
Demand 1/3	1/3, 2/3	1/3, 2/3	1/3, 2/3
Demand 1/2	1/2, 1/2	1/2, 1/2	0, 0
Demand 2/3	2/3, 1/3	0, 0	0, 0

Evolutionary Analysis of the Ultimatum Game

A strategy is an ordered pair $\langle a, b \rangle$ where a is the demand (when the player is a Proposer) and b in the minimum acceptable (when the player is a Responder).

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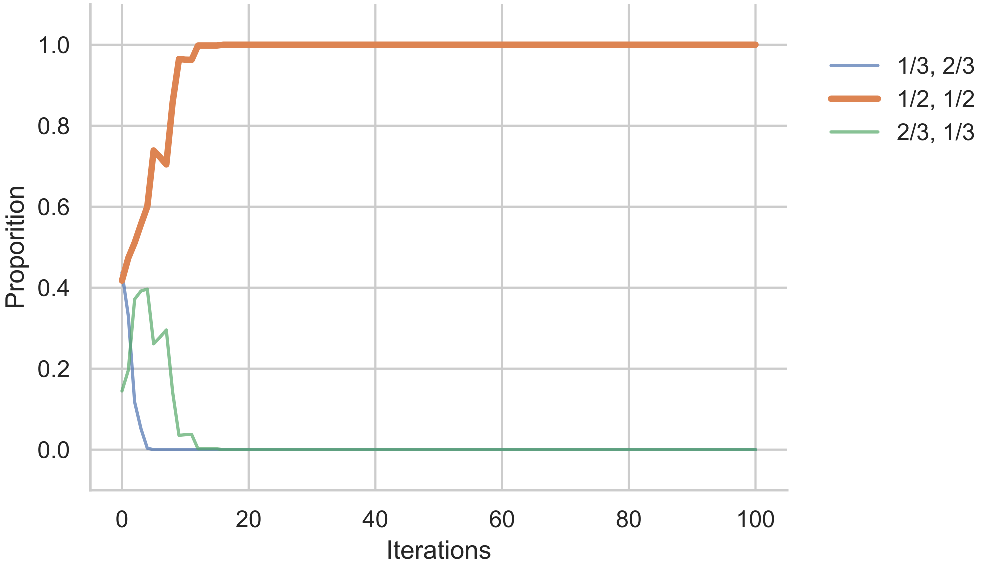
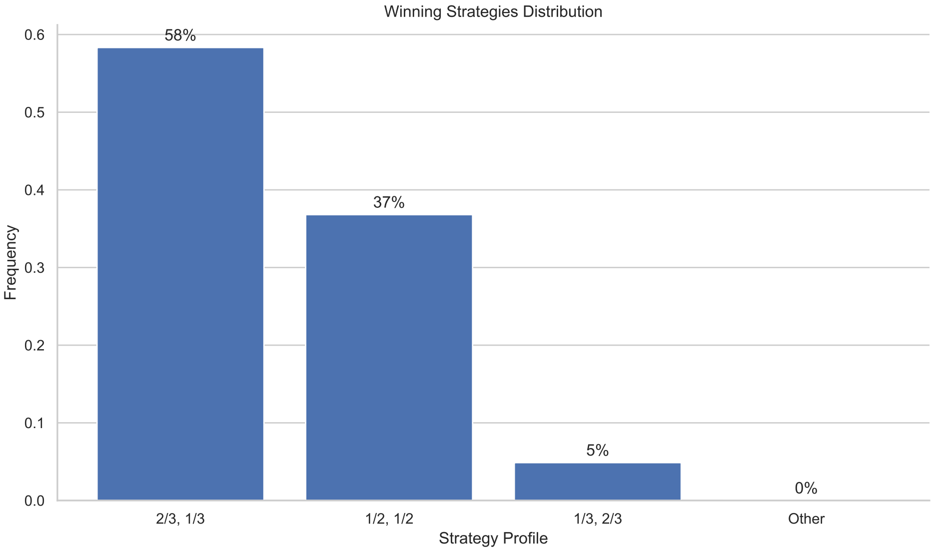
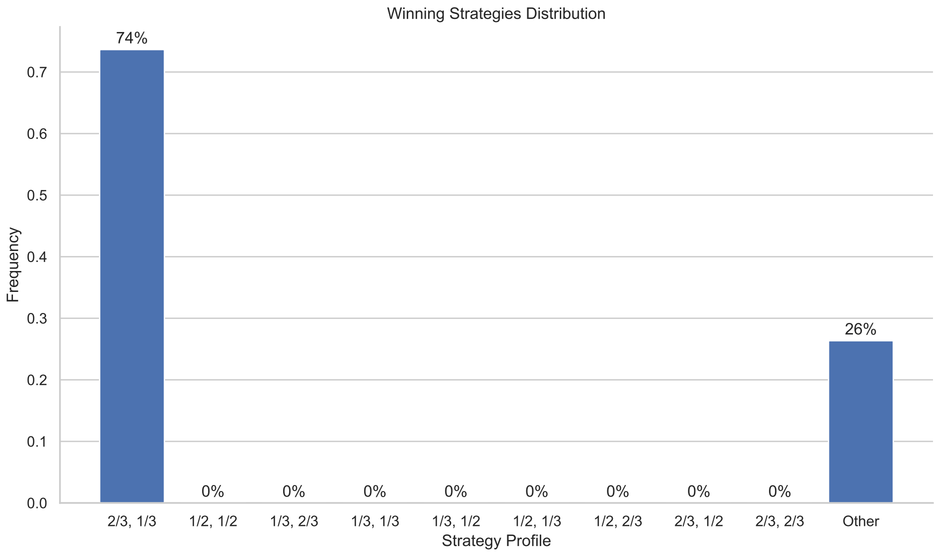
Evolutionary Analysis of the Ultimatum Game

A strategy is an ordered pair $\langle a, b \rangle$ where a is the demand (when the player is a Proposer) and b in the minimum acceptable (when the player is a Responder).

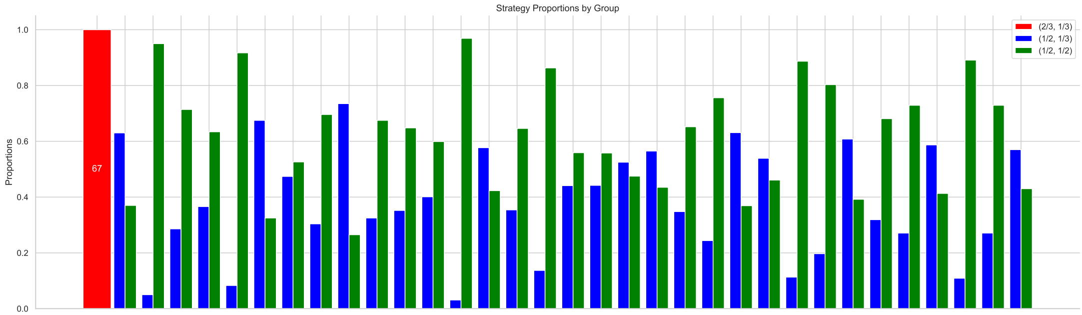
We assume that individuals are randomly paired from the population; that the decision as to which individual is to play which role is made at random; and that the payoffs are in terms of evolutionary fitness. Because a strategy determines what a player will do in each role, we can now calculate the expected fitness for any of the strategies that results from an encounter with any of the other strategies.


1.0
0.8
0.6
0.4
0.2
0.0
Proportion
0
20
40
60
80
100
Iterations
1/3, 2/3
1/2, 1/2
2/3, 1/3


Winning Strategies Distribution
Frequency
0.0
0.1
0.2
0.3
0.4
0.5
0.6
58%
37%
5%
0%
2/3, 1/3
1/2, 1/2
1/3, 2/3
Other
Strategy Profile



Winning Strategies Distribution
74%
26%
0%
0%
0%
0%
0%
0%
0%
0%
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Frequency
2/3, 1/3
1/2, 1/2
1/3, 2/3
1/3, 1/3
1/3, 1/2
1/2, 1/3
1/2, 2/3
2/3, 1/2
2/3, 2/3
Other
Strategy Profile


Strategy Proportions by Group
Proportions
1.0
0.8
0.6
0.4
0.2
0.0
67
(2/3, 1/3)
(1/2, 1/3)
(1/2, 1/2)

[U]ltimatum game behavior does not arise simply in the context of repeated ultimatum games.

[U]ltimatum game behavior does not arise simply in the context of repeated ultimatum games. Rather experimental behavior may be explained by cultural norms that the subjects, perhaps unconsciously, apply. Such social norms evolve for large classes of social interactions that are frequently encountered.

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Thus, in the ultimatum game, [the Responder]

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- ...could see it as an ultimatum game in which the other player was trying to take unfair advantage and apply the rule "Don't accede to unfair offers in the ultimatum game," or
- ...could see it as a general bargaining game and apply a rule evolved for a class of bargaining games, or

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- ...could see it as an ultimatum game in which the other player was trying to take unfair advantage and apply the rule “Don’t accede to unfair offers in the ultimatum game,” or
- ...could see it as a general bargaining game and apply a rule evolved for a class of bargaining games, or
- ...could see it as a game sending a signal prior to subsequent interactions.

K. Zollman (2008). *Explaining fairness in complex environments*. Philosophy, Politics, and Economics, 7(1), pp. 81 - 98.