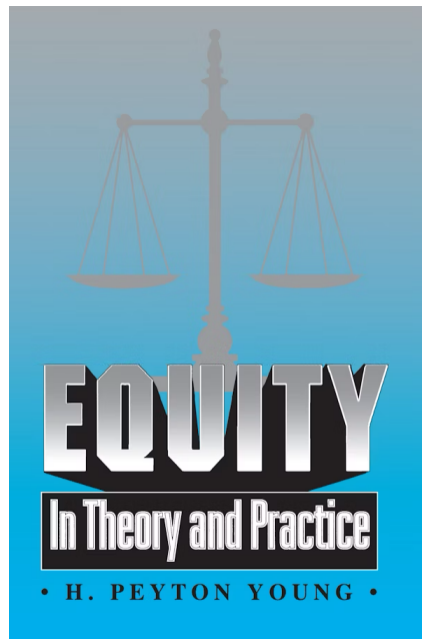


# PHPE 308M/PHIL 209F

## Fairness

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An **allocation** or **distribution** is an assignment of the objects to specific individuals.

- ▶ Allocation is not the same as exchange. An allocation is about who gets a good or who bears a burden. Exchange involves many voluntary, decentralized transactions, and can only occur after the goods and burdens have been allocated.

Three classic approaches to fair division:

1. Proportionality
2. Classical utilitarianism
3. Rawls's maximin principle.

# Proportionality

Aristotle's equity principle: goods should be divided in proportion to each claimant's contribution.

# Proportionality

We must have some way to measure the contribution of each claimant on a cardinal scale.

Sometimes such a measure is natural, for example, the amount of time each worker put into a joint effort. However, in a divorce proceeding, for example, how does one ascertain the relative contribution that husband and wife made to their joint estate or to rearing their children?

# Proportionality

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For example, if A has fought twice as long in the army as B and only one of them can be discharged, should A get twice as many chances at being discharged as B? Or is it fairer simply to discharge A first because he fought longer than B?

# Utilitarianism

Classical Utilitarianism: goods should be distributed so as to maximize the total welfare of the claimants (the greatest good for the greatest number).

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2. Even if we could devise some method for comparing individual utilities, it is not clear that the utilitarian principle is ethically sound, since it might require imposing great harm on a few in order to confer a small benefit on the many.

# Maximin Principle

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It is not clear that the maximin principle satisfies our intuitions about justice:

Is it just to impose serious inconveniences on almost everyone in society in order to raise the opportunities, the income, or the self-respect of the least fortunate by a miniscule amount?

# Allocation Problems

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For example, which patients should receive organs for transplantation? Which soldiers in the Army should be allowed to go home first? Who gets the corner office? Which occupant of the lifeboat gets eaten when the food runs out?

**Forced Equality:** When equal treatment is the paramount consideration, the good may be given to no one. No differences in entitlement are recognized; everyone must receive the same amount. When the good or burden is indivisible, meeting this ideal can be quite wasteful.

**Lotteries:** One way to avoid the inefficiency of forced equality is to give everyone an equal chance at getting the good or bearing the burden. Then everyone is treated equally before the fact, though not afterward. Lotteries usually involve equal chances, but there is no reason why some claimants should not be given more chances than others.

**Rotation:** In some circumstances it is more appealing to divide an indivisible good by taking turns. Unlike lotteries, there is no tension here between *ex ante* and *ex post* fairness. On the other hand, the sharing process can substantially change the character of the good itself. Half custody of a child is not the same thing as half a child.

**Compensation:** Another device for resolving indivisibilities is to compensate those who do not get the good (or, in the case of a burden, to compensate those who do).

**Queuing:** A more standard approach to allocating indivisibles is to give them out to those who are first in line. Queuing is akin to a lottery, since it is partly a matter of chance who gets in line first. But not wholly: one can be first by spending the most time in line. In this sense queuing is like an auction: those who bid the most time get the good.

**Priority Lists:** A more general form of the waiting list is the priority list, in which claimants are ranked according to some measure of need, desert, contribution, seniority, or (more typically) a combination of factors. A priority list reflects an equity judgment about who deserves the good most.

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Priority lists are probably the most widely used of any of the above methods for allocating indivisible goods. They are simple in concept, they have the advantage of allocating the good itself rather than something else (like a timeshared good), and they make the basis for the allocation explicit.

# Kidney Transplants

In recognition of the growing difficulty of matching kidneys with transplant patients, Congress passed the National Organ Transplant Act in 1984.

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In recognition of the growing difficulty of matching kidneys with transplant patients, Congress passed the National Organ Transplant Act in 1984. This legislation called for “a national system, through the use of computers and in accordance with established medical criteria, to match organs and individuals included in the recipient list.” This Act and its subsequent amendments led to the establishment of the Organ Transplantation and Procurement Network, which is operated by the United Network of Organ Sharing (UNOS).

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The kidney formula is based on a **priority list** in which rank-order is determined by assigning point values to various pertinent factors. These factors and their weights were determined after extensive discussions by committees consisting of medical experts, ethicists, representatives of patient groups, and members of the general public.

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There are three broad factors:

1. efficacy — the likelihood that the transplant will be a success;
2. need — the lack of alternatives such as dialysis;
3. disadvantage — patients who are difficult to match should be given a handicap;

The goal of efficacy suggests that kidneys should be allocated where they will yield the maximum benefit.

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One measure of efficacy is the expected gain in useful years of life. As this criterion is rather difficult to pin down, however, indicators of the probability of success are used. Studies have shown, for example, that the more antigens that are matched between donor and recipient the higher the likelihood of success.

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The original UNOS formula awarded **two points** for each of the six possible antigen matches between donor and patient.

In addition, a bonus of up to **six points** was awarded if the logistics of getting the kidney to the patient were favorable.

The second consideration is medical urgency. Typically this situation arises when a patient cannot remain on dialysis because all of the available dialysis sites have been used up. In this case the patient receives a bonus of **ten points**.

The third consideration is to try to avoid lengthy waits for a kidney.

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Some renal patients are inherently disadvantaged because they are highly *sensitized*, that is, they have antibodies against a high proportion of the rest of the population, and hence against most prospective donors. Such patients are given a handicap to compensate them for biological “bad luck.”

They are awarded **1 point for each 10 percent of the general population against which they have antibodies**. Thus, if a patient is sensitized against 80 percent of the population, he would receive 8 points.

In addition, a patient earns points the longer he has been waiting (whether or not he is highly sensitized).

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If there are  $n$  patients, then the  $k$ th person in line would get a score of

$$10 \frac{(n - k + 1)}{n}$$

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If there are  $n$  patients, then the  $k$ th person in line would get a score of

$$10 \frac{(n - k + 1)}{n}$$

So, if there are 5 persons on the list, the first person receives  $10(5 - 1 + 1)/5 = 10$  points, the second person receives  $10(5 - 2 + 1)/5 = 8$  points, and so on.

# The Point System

- ▶ **2 points** for each of the six possible antigen matches between donor and patient.
- ▶ **Up to 6 points** for the logistics of getting the kidney to the patient.
- ▶ **10 points** for medical urgency.
- ▶ If there are  $n$  patients, then the  $k$ th person in line would get

$$10^{\frac{(n - k + 1)}{n}} \text{ points.}$$

**TABLE 2.3****Characteristics of Five Patients in a Given Blood Group for Determining Priority**

<i>Patient</i>	<i>Months</i>	<i>Antigens Matched</i>	<i>Sensitization (%)</i>	<i>Logistics</i>	<i>Urgency</i>
A	5	2	10	0	0
B	4.5	2	20	0	0
C	4	0	0	5	0
D	2	3	60	0	0
E	1	6	90	0	0

Patient	Line	Antigens	Sensitization	Log/Urg	Points
<i>A</i>	$k_1$	<i>A</i>	<i>S</i>	<i>L</i>	$w_{time} + 2 \times A + S/10 + L$
<i>B</i>	$k_2$	<i>A</i>	<i>S</i>	<i>L</i>	$w_{time} + 2 \times A + S/10 + L$
<i>C</i>	$k_3$	<i>A</i>	<i>S</i>	<i>L</i>	$w_{time} + 2 \times A + S/10 + L$
<i>D</i>	$k_4$	<i>A</i>	<i>S</i>	<i>L</i>	$w_{time} + 2 \times A + S/10 + L$
<i>E</i>	$k_5$	<i>A</i>	<i>S</i>	<i>L</i>	$w_{time} + 2 \times A + S/10 + L$

$$w_{time} = 10^{\frac{(5 - k + 1)}{5}}$$

Patient	Line	Antigens	Sensitization	Log/Urg	Points
<i>A</i>	1	2	10	0	$10 + 4 + 1 + 0 = 15$
<i>B</i>	2	2	20	0	$8 + 4 + 2 + 0 = 14$
<i>C</i>	3	0	0	5	$6 + 0 + 0 + 5 = 11$
<i>D</i>	4	3	60	0	$4 + 6 + 6 + 0 = 16$
<i>E</i>	5	6	90	0	$2 + 12 + 9 + 0 = 23$

$$w_{time} = 10^{\frac{(5 - k + 1)}{5}}$$

Suppose that there are two available kidneys. The priority lists implies that the kidneys should go to patients *E* and *D* in that order.

Patient	Line	Antigens	Sensitization	Log/Urg	Points
<i>A</i>	1	2	10	0	$10 + 4 + 1 + 0 = 15$
<i>B</i>	2	2	20	0	$8 + 4 + 2 + 0 = 14$
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Suppose that the two kidneys do not become available at the same time. (This might happen because one of them is diverted to another transplant center, for example.) The first kidney clearly goes to  $E$ , since  $E$  has highest priority.

Patient	Line	Antigens	Sensitization	Log/Urg	Points
<i>A</i>	<i>k</i> <sub>1</sub>	<i>A</i>	<i>S</i>	<i>L</i>	$w_{time} + 2 \times A + S/10 + L$
<i>B</i>	<i>k</i> <sub>2</sub>	<i>A</i>	<i>S</i>	<i>L</i>	$w_{time} + 2 \times A + S/10 + L$
<i>C</i>	<i>k</i> <sub>3</sub>	<i>A</i>	<i>S</i>	<i>L</i>	$w_{time} + 2 \times A + S/10 + L$
<i>D</i>	<i>k</i> <sub>4</sub>	<i>A</i>	<i>S</i>	<i>L</i>	$w_{time} + 2 \times A + S/10 + L$
<i>E</i>	<i>k</i> <sub>5</sub>	<i>A</i>	<i>S</i>	<i>L</i>	$w_{time} + 2 \times A + S/10 + L$

$$w_{time} = 10^{\frac{(4 - k + 1)}{4}}$$

Patient	Line	Antigens	Sensitization	Log/Urg	Points
<i>A</i>	1	2	10	0	$10 + 4 + 1 + 0 = 15$
<i>B</i>	2	2	20	0	$7.5 + 4 + 2 + 0 = 13.5$
<i>C</i>	3	0	0	5	$5 + 0 + 0 + 5 = 10$
<i>D</i>	4	3	60	0	$2.5 + 6 + 6 + 0 = 14.5$

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Patient	Line	Antigens	Sensitization	Log/Urg	Points
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<i>D</i>	4	3	60	0	$2.5 + 6 + 6 + 0 = 14.5$

According to the Priority Order, *A* gets the kidney!

**TABLE 2.4****Point Values for Five Patients Before and After Patient E Receives a Kidney**

<i>Patient</i>	<i>Months Waiting</i>		<i>Antigens</i>	<i>Sensitization</i>	<i>Log &amp; Urg</i>	<i>Total</i>	
	<i>Before E Deleted</i>	<i>After E Deleted</i>				<i>Before</i>	<i>After</i>
A	10	10	4	1	0	15	15
B	8	7.5	4	2	0	14	13.5
C	6	5	0	0	5	11	10
D	4	2.5	6	6	0	16	14.5
E	2	—	12	9	0	23	—

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Now consider the individuals remaining after  $E$  has gone home. The point values for waiting time change, because they depend on the number of people in the queue as well as on their relative position.

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Now consider the individuals remaining after  $E$  has gone home. The point values for waiting time change, because they depend on the number of people in the queue as well as on their relative position.

Therefore, if another kidney of the same type comes along, then  $A$  would receive it ahead of  $D$ !

This solution seems nonsensical. Why should the priority of two patients, for the same type of kidney, switch depending on who else is in line? If a priority system means anything, then surely it tells us which of two claimants has priority over the other, irrespective of who else is a claimant.

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The UNOS formula was adapted to avoid this type of situations.