PHPE 308M/PHIL 209F Fairness

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A	B
a	a
b	*b*
c	c
d	*d*
e	e
f	*f*

The picking sequence *ABABAB* produces the allocation:

A: *a, c, e*
B: *b, d, f*

The allocation is efficient, but not envy-free.

A	B
a	*f*
b	*e*
c	*d*
d	c
e	b
f	a

The picking sequence *ABABAB* produces the allocation:

A: *a*, *b*, *c*
B: *f*, *e*, *d*

The allocation is efficient and envy-free.

Picking procedures always guarantee Pareto-optimal allocations. In fact, an allocation is Pareto-optimal if and only if it can be obtained from a sequence of sincere choices by the players.

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For different social welfare functions and preference distributions, what are the optimal picking sequences?

S. Bouveret and J. Lang (2011). *A general elicitation-free protocol for allocating indivisible goods*. In: Proceedings of the 22nd international joint conference on artificial intelligence (IJCAI), AAAI, pp. 73 - 78.

A problem with picking sequences...

A	B
a	b
b	c
c	d
d	a

The picking sequence *ABAB* produces the allocation **assuming that the players choose sincerely**:

A: a, c
B: b, d

A	B
a	b
b	c
c	d
d	a

A can improve her allocation by selecting *insincerely*: If A chooses b in the first round, then B will choose c, resulting in the allocation:

A: a, b
B: c, d

Strategy-Proofness

An important property of a procedure is **strategy-proofness**: telling the truth is a dominant strategy for all players.

Why Envy-Freeness?

R. Sugden (1984). *Is Fairness Good? A Critique of Varian's Theory of Fairness*. Noûs, 18(3), pp. 505-511.

H. R. Varian (1975). *Distributive Justice, Welfare Economics, and the Theory of Fairness*. Philosophy and Public Affairs 4, pp. 223-247.

Envy: to envy someone is not to feel ill-will towards him, or to experience disutility when reflecting on his good fortune; it is simply to prefer what he has to what one has oneself. An allocation is **envy-free** if no person envies any other person.

Pareto-efficient: no other feasible allocation exists such that at least one person prefers the latter to the former and no one prefers the former to the latter.

Fairness Principle: If an Pareto-efficient and envy-free allocation is feasible, then no allocation that is not Pareto-efficient and envy-free should be chosen.

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What is the justification for this principle?

Varian on Fairness

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A second claim is that envy-free allocations ensure 'equity':

1. The definition treats all persons symmetrically
2. The definition of *envy-freeness* is just a formal definition that is not meant to reflect ordinary usage.
3. *Envy-freeness* 'is of interest in formalizing certain ordinary concepts of equity'.

But what follows is not a moral argument, nor an analysis of how people ordinarily use the concept of equity. It is a technical argument, designed to show that this concept of fairness is 'quite operational.'

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This may provide an answer to the practical question, 'Could this concept of fairness be used to guide social choices?'; but Varian never answers the equally important moral question, 'Why should it be so used?'

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He seems to rely on his readers sharing his intuitions that Pareto-efficiency is good and that envy-freeness is equivalent to equity.

Paretian Welfare

1. To the extent that we are concerned with a person's welfare, we must concern ourselves only with what he *wants* (rather than, for example, with what we think is good for him).
2. The welfare of society depends only on the welfare of the individuals who comprise it.
3. If one person's welfare increases, other things remaining equal, then social welfare increases....'social welfare' is, in effect, being used as a synonym for 'the good of society, all things considered.'

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This can be reduced to a single maxim: *as far as social choice is concerned, all that matters is the satisfaction of wants.*

Pareto inefficient outcomes are *dominated* along the n-dimensions of the individuals 'wants'.

			$\cdots$		$\cdots$	
A	1	10	$\cdots$	7	$\cdots$	6
			$\cdots$		$\cdots$	
B	1	10	$\cdots$	5	$\cdots$	3
			$\cdots$		$\cdots$	
Individuals	1	2	$\cdots$	k	$\cdots$	n

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Individuals	1	2	$\cdots$	k	$\cdots$	n

Inefficient allocations ought not to be chosen: Do not choose the allocation *B* since *A* is clearly superior in the sense that it more effectively satisfies the wants of the *n* individuals.

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Two different rules:

- ✗ Choose any allocation so long as it is Pareto-efficient.
- ✓ Don't choose a Pareto-inefficient allocation.

The second rule is the rule that Varian wants to call 'certainly reasonable.'

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▶ The rule of not choosing Pareto-inefficient allocations is only as reasonable as the principle from which it derives: that nothing matters except the satisfaction of wants.

▶ **But if nothing matters except the satisfaction of wants, there seems to be no reason for valuing envy-freeness.**

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  - ▶ If you have cream with your pie while I don't, I envy you. If instead we both go without cream, there is no envy.
  - ▶ But I want cream just as much in either case; your forgoing it eliminates my envy but it does nothing to satisfy my want.

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If envy-freeness is good, it is because something matters apart from the satisfaction of wants.

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This is not everyone's idea of a good society, but it is some people's. For such people equality or perhaps more accurately harmony-is a dimension of social welfare in its own right, and an envy-free society is one that has achieved the maximum possible degree of harmony.

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This is a perfectly coherent position to take, but it is inconsistent with the Paretian theory

Varian's principle of fairness amounts, in effect, to the following rule:

> First assume that all that matters is want-satisfaction, and rule out any allocations that would be clearly inferior if that assumption were true; then assume that all that matters is harmony and rule out any allocations that would be clearly inferior if that assumption were true.

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First assume that all that matters is want-satisfaction, and rule out any allocations that would be clearly inferior if that assumption were true; then assume that all that matters is harmony and rule out any allocations that would be clearly inferior if that assumption were true.

Claim: But since these two assumptions contradict one another, any attempt to justify a rule based on both of them seems doomed to failure.

Sugden's Example

Me	You
jam roll with custard	apple pie with cream
jam roll	apple pie
apple pie with cream	jam roll with custard
apple pie	jam roll

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jam roll with custard	apple pie with cream
jam roll	apple pie
apple pie with cream	jam roll with custard
apple pie	jam roll

Feasible allocations:

- ▶ A_1 gives us both jam roll
- ▶ A_2 gives me jam roll and you jam roll with custard
- ▶ A_3 gives us both apple pie.

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A_1 and A_3 are envy-free but A_2 is not (I envy your custard.)

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A_2 and A_3 are Pareto-efficient but A_1 is not.

A_1 and A_3 are envy-free but A_2 is not (I envy your custard.)

So, we must choose the allocation A_3 since it is the only envy-free and Pareto-efficient allocation.

Me	You
jam roll with custard	apple pie with cream
jam roll	apple pie
apple pie with cream	jam roll with custard
apple pie	jam roll

But what grounds are there for saying that A_3 is better than A_1 or A_2 (both give me a jam roll)?

Me	You
jam roll with custard	apple pie with cream
jam roll	apple pie
apple pie with cream	jam roll with custard
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The satisfaction of my wants is one dimension of social welfare, and I prefer both A_1 and A_2 to A_3. So the simple proposition that want satisfaction and harmony both matter cannot provide a sufficient reason for choosing A_3.

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To justify this choice, some argument must be made about the relative importance of satisfying your want for apple pie as opposed to my want for jam roll, and this is exactly the kind of argument that Varian claims to avoid.

Me	You
jam roll with custard	apple pie with cream
jam roll	apple pie
apple pie with cream	jam roll with custard
apple pie	jam roll

Me	You
jam roll with custard	apple pie with cream
jam roll	apple pie
apple pie with cream	jam roll with custard
apple pie	jam roll

Feasible allocations:

- ▶ A_1 gives us both jam roll
- ▶ ~~A_2 gives me jam roll and you jam roll with custard~~
- ▶ A_3 gives us both apple pie.
- ▶ A_4 gives me apple pie and you apple pie with cream.

Me	You
jam roll with custard	apple pie with cream
jam roll	apple pie
apple pie with cream	jam roll with custard
apple pie	jam roll

Feasible allocations:

- ▶ A_1 gives us both jam roll
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The degree to which an allocation satisfies wants and the degree to which it inspires envy are both surely independent of whether other allocations happen to be feasible or not.