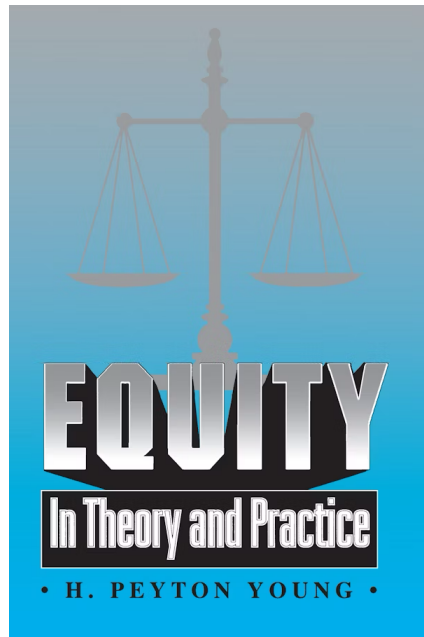


PHIL 408Q/PHPE 308D

Fairness

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An **allocation** or **distribution** is an assignment of the objects to specific individuals.

- ▶ Allocation is not the same as exchange. An allocation is about who gets a good or who bears a burden. Exchange involves many voluntary, decentralized transactions, and can only occur after the goods and burdens have been allocated.

Priority Lists: A more general form of the waiting list is the priority list, in which claimants are ranked according to some measure of need, desert, contribution, seniority, or (more typically) a combination of factors. A priority list reflects an equity judgment about who deserves the good most.

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Priority lists are probably the most widely used of any of the above methods for allocating indivisible goods. They are simple in concept, they have the advantage of allocating the good itself rather than something else (like a timeshared good), and they make the basis for the allocation explicit.

Claimant Type: The type of a claimant is a complete description of the claimant for purposes of the allocation, and determines the extent of a claimant's entitlement to the good.

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Consistency: A zero-one allocation criterion is pairwise consistent if, whenever two types τ and τ' share one unit, then they always share it in the same way — either τ gets it exclusively, or τ' gets it exclusively, or there is a tie between them. The decision does not depend on what other claimants are present or how much they get.

Standard of Comparison: A standard of comparison is a list of all types, ordered from highest to lowest priority. Distinct types may have equal priority, in which case we say they are *on a par*.

Priority Method: The priority method based on a given standard distributes the available units to the claimants who have highest priority.

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If there are only two units to distribute, the priority criterion would give them either to *A* and *B*, or to *A* and *C*. The tie could be resolved by a chance device, such as tossing a fair coin.

Priority methods are the only allocation methods that satisfy the two basic principles of equity:

A zero-one allocation criterion is impartial and pairwise consistent if and only if it is a priority method.

Determining the Priority Method

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This is the *opinion aggregation* or *social choice problem*.

Fix a specific class of allocation problems, such as kidney transplants or army demobilization, and let T be the (finite) set of types that are relevant to the good being distributed.

Imagine that a group of individuals is asked to rank the types according to some notion of priority. That is, each individual gives his opinion as to the most appropriate ordering. In some cases the opinions will be highly correlated and there is no particular difficulty in identifying the consensus ordering. In other situations there is no clear consensus, and one must try to balance the various opinions fairly.

	<i>Efficacy</i>	<i>Urgency</i>	<i>Time Waiting</i>
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B	5	1	0.5
C	20	5	3

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Opinions of Sixty Respondents about the
Relative Priority of Three Patients

<i>Number of Committee Members with the Given Opinion</i>				
<i>13</i>	<i>10</i>	<i>6</i>	<i>13</i>	<i>18</i>
A	A	B	B	C
C	B	A	C	B
B	C	C	A	A

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TABLE 2.6

Vote Matrix Corresponding to the
Opinions in Table 2.5

	<i>A</i>	<i>B</i>	<i>C</i>	<i>Row Sum</i>
A	0	23	29	52
B	37	0	29	66
C	31	31	0	62

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Borda ranking: Given a list of voter opinions about the ranking of the alternatives types, the **Borda score** of each alternative is the total number of types that are ranked below it, summed over all individual opinions. The Borda rule orders the the alternatives according to their Borda scores.

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Condorcet's idea: Choose the ranking(s) that are *supported by the maximum number of pairwise votes*.

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Condorcet's Criterion: Given a set of types and a ranking of these types by each member of a group, a Condorcet ranking is one in which the pairwise assertions of priority as supported by the maximum number of individuals in the group.

The Condorcet ranking is *C B A*.

The Paradox of Voting

TABLE 2.7

**A Cyclic Majority: A Defeats B,
B Defeats C, C Defeats A**

6	5	2
A	B	C
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Problem: There is no majority alternative.

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Problem: There is no majority alternative.

Examples like this show that there is no obvious way to extend simple majority rule to situations with more than two alternatives. No matter how we rank the alternatives A , B , and C , one of them will have a strict majority over some other alternative that is ranked higher. This observation is at the heart of a key result in voting theory known as *Arrow's Impossibility Theorem*.

Arrow's Impossibility Theorem

There is no method for aggregating individual rankings into a single consensus ordering that meets the following three conditions.

1. Unanimity: if all voters rank some alternative A above another alternative B , then A is ranked above B in the consensus.
2. Nondictatorship: the consensus ranking is not dictated by the same individual in all situations.
3. Independence of irrelevant alternatives: the relative rank of each pair of alternatives in the consensus order depends only on the individual opinions regarding that pair.

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This is not to suggest, however, that no aggregation scheme should be used. Rather, we would argue that opinion surveys of affected groups have a legitimate place in determining priorities, and that an aggregation scheme may be necessary to balance diverse responses in an objective way.

In theory there are many ways of allocating an indivisible good fairly among competing claimants, including lotteries, rotation, and compensation schemes. In practice, however, society often allocates a scarce indivisible good by appealing to some notion of *priority* among the claimants. Moreover, there is justification for this approach in that priority methods are the only ones that allocate the good both impartially and consistently over different situations.