PHIL 408Q/PHPE 308D
Fairness

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Allocations

Suppose that X is a set of items and I is a set of agents, or players.

An **allocation** assigns to each agent in I some of the items from X such that no item is allocated to more than one agent.

- An allocation is **complete** provided that all items are allocated.
- An allocation is **balanced** provided that the agents receive the same number of items.

Preferences

1. Each player has a preference over the set of items.

2. Players cannot compensate each other with side payments—the division is only of the indivisible items.

3. All players have positive values for every item.

4. A player prefers one set S of items to a different set T if (i) S has as many items as T and (ii) for every item t in T and not in S, there is a distinct item s in S and not T that the player prefers to t.

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Fairness Conditions

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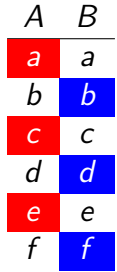
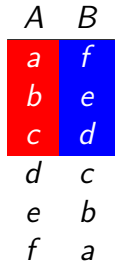
- **Efficiency (Pareto-Optimality):** there is no other allocation that is at least as good for all players and strictly better for at least one player.



The picking sequence $ABABAB$ produces the allocation:

A: a, c, e
B: b, d, f

The allocation is efficient, but not envy-free.



The picking sequence $ABABAB$ produces the allocation:

A: a, b, c
B: f, e, d

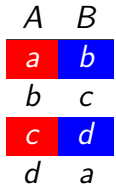
The allocation is efficient and envy-free.

Picking procedures always guarantee Pareto-optimal allocations. In fact, an allocation is Pareto-optimal if and only if it can be obtained from a sequence of sincere choices by the players.

S. Brams and D.L. King (2005). *Efficient fair division: help the worst off or avoid envy?*. Rationality and Society,17, pp. 387 - 421.

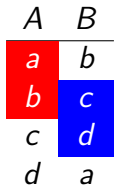
For different social welfare functions and preference distributions, what are the optimal picking sequences?

S. Bouveret and J. Lang (2011). *A general elicitation-free protocol for allocating indivisible goods*. In: Proceedings of the 22nd international joint conference on artificial intelligence (IJCAI), AAAI, pp. 73 - 78.



The picking sequence $ABAB$ produces the allocation assuming that the players choose *sincerely*:

A: a, c
B: b, d



A can improve her allocation by selecting *insincerely*: If A chooses b in the first round, then B will choose c, resulting in the allocation:

A: a, b
B: c, d

An important property of a procedure is **strategy-proofness**: telling the truth is a dominant strategy for all players.

Adjusted Winner

Adjusted winner (*AW*) is an algorithm for dividing n divisible goods among two people (invented by Steven Brams and Alan Taylor).

For more information see

- *Fair Division: From cake-cutting to dispute resolution* by Brams and Taylor, 1998
- *The Win-Win Solution* by Brams and Taylor, 2000
- https://www.nyu.edu/projects/adjustedwinner

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Adjusted winner (*AW*) is an algorithm for dividing n divisible goods among two people (invented by Steven Brams and Alan Taylor).

Point Assignment: Both Ann and Bob distribute 100 points among the three items

Winner Take All: The person who assigned the most points is given that good

Equitability Adjustment: Transfer all or part of the goods from the person with the most points until both receive the same number of points

Item	Ann	Bob
A		
B		
C		

Suppose Ann and Bob are dividing three goods $\{A, B, C\}$

Item	Ann	Bob
A	5	4
B	65	46
C	30	50
Total	100	100

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Item	Ann	Bob
A	5	0
B	65	0
C	0	50
Total	70	50

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Find the item whose ratio is closes to 1: $65/46 \geq 5/4 \geq 1 \geq 30/50$

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A	5	4
B	65	46
C	30	50
Total	100	100

Item	Ann	Bob
A	0	4
B	65	0
C	0	50
Total	65	54

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Still not equal, so give (some of) B to Bob: $65p = 100 - 46p$ yielding $p = \frac{100}{111} = 0.901$

Item	Ann	Bob
A	5	4
B	65	46
C	30	50
Total	100	100

Item	Ann	Bob
A	0	4
B	58.56	4.56
C	0	50
Total	58.56	58.56

Suppose Ann and Bob are dividing three goods $\{A, B, C\}$

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Easy Observations

- For two-party disputes, proportionality and envy-freeness are equivalent.
- *AW* only produces **equitable** allocations (equitability is essentially built in to the procedure).
- *AW* produces allocations in which at most one good is split.

Adjusted Winner is Fair

Theorem (Brams and Taylor) *AW produces allocations that are efficient, equitable and envy-free (with respect to the announced valuations).*

Strategizing

In Adjusted Winner, can the people improve their allocation by misrepresenting their preferences?

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Yes

Strategizing: Example

Item	Ann	Bob
Matisse	75	25
Picasso	25	75

Ann will get the Matisse and Bob will get the Picasso and each gets 75 of his or her points.

Strategizing: Example

Suppose Ann knows Bob's preferences, but Bob does not know Ann's.

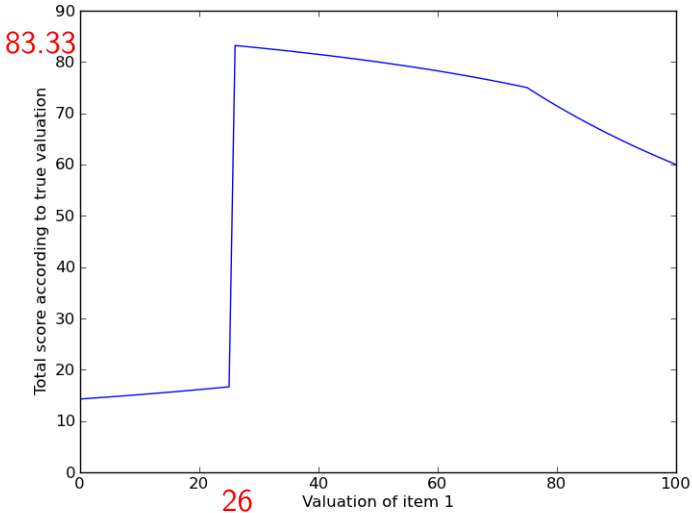
Item	Ann	Bob
M	75	25
P	25	75

Item	Ann	Bob
M	26	25
P	74	75

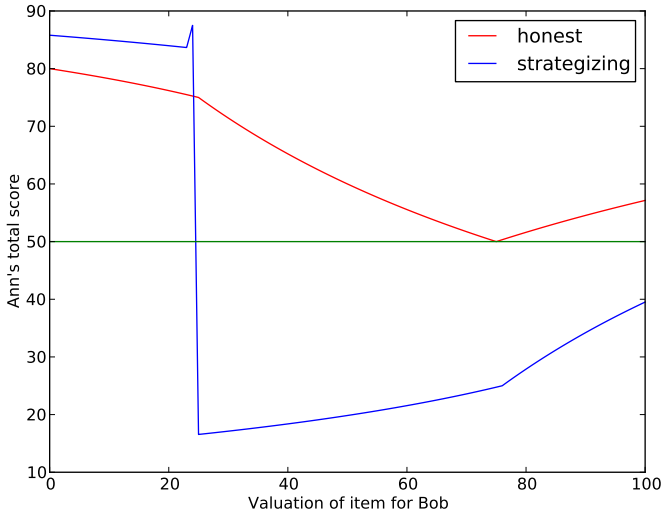
So Ann will get M plus a portion of P.

According to Ann's announced allocation, she receives 50.33 points

According to Ann's actual allocation, she receives $75 + 0.33 * 25 = 83.33$ points.


83.33
26
Total score according to true valuation
Valuation of item 1
0
10
20
30
40
50
60
70
80
90
0
20
40
60
80
100

However, while honesty may not always be the best policy it is the only **safe** one, i.e., the only one which will guarantee 50%.


honest
strategizing
Ann's total score
Valuation of item for Bob
90
80
70
60
50
40
30
20
10
0
20
40
60
80
100

Strategizing: Example

Suppose *both* players know each other's preferences but neither knows that the other knows their own preference.

Item	Ann	Bob
M	75	25
P	25	75

Item	Ann	Bob
M	26	74
P	74	26

Each will get 74 of his or her announced points, but each one is really getting only 25 of his or her *true* points.

Strategizing: Example

Suppose *both* players know each other's preferences. Moreover, Ann knows that Bob knows her preference and Bob doesn't know that Ann knows.

Item	Ann	Bob
M	26	74
P	74	26

Item	Ann	Bob
M	73	74
P	27	26

What happens as the level of knowledge increases?

R. Sugden (1984). *Is Fairness Good? A Critique of Varian's Theory of Fairness*. Noûs, 18(3), pp. 505-511.

Envy here is to be understood in the sense that Rawls calls 'benign envy': to envy someone is not to feel ill-will towards him, or to experience disutility when reflecting on his good fortune; it is simply to prefer what he has to what one has oneself.

An allocation is **Pareto-efficient** if no other feasible allocation exists such that at least one person prefers the latter to the former and no one prefers the former to the latter.

Varian's principle of fairness: if a Varian-fair allocation is feasible, then no allocation that is not Varian-fair should be chosen.

I shall try to show that this principle cannot be justified in terms of any coherent and plausible system of values.

Varian on Fairness

All he has to say about Pareto-efficiency is that 'the criterion of Pareto efficiency is certainly reasonable: if there is some way to make everyone better off, why not do it?'. Of his definition of 'equity' as envy-freeness, he first notes that it, like many other possible definitions, treats all persons symmetrically. Next he says that it is just a formal definition that is not meant to reflect ordinary usage, and then that he hopes to show that his definition 'is of interest in formalizing certain ordinary concepts of equity'.

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This may provide an answer to the practical question, 'Could this concept of fairness be used to guide social choices?'; but Varian never answers the equally important moral question, 'Why should it be so used?' He seems to rely on his readers sharing his intuitions that Pareto-efficiency is good and that envy-freeness is equivalent to equity.

Paretian Welfare

1. To the extent that we are concerned with a person's welfare, we must concern ourselves only with what he *wants* (rather than, for example, with what we think is good for him).
2. The welfare of society depends only on the welfare of the individuals who comprise it.
3. If one person's welfare increases, other things remaining equal, then social welfare increases....'social welfare' is, in effect, being used as a synonym for 'the good of society, all things considered.'

Without too much loss of precision, their content can be compressed into a single maxim: *as far as social choice is concerned, all that matters is the satisfaction of wants*.