PHIL 408Q/PHPE 308D
Fairness

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Cut-and-choose

Two Players

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This strategy creates an **envy-free** and **efficient** allocation, but it is not necessarily **equitable**.

Example

Suppose that the cake is half chocolate and have vanilla.
Ann values the vanilla half twice as much as the chocolate half:

$$v_A(x) = \begin{cases} 4/3 & x \in [0, 1/2] \\ 2/3 & x \in (1/2, 1] \end{cases}$$

Bob values both sides equally:

$$v_B(x) = \begin{cases} 1 & x \in [0, 1/2] \\ 1 & x \in (1/2, 1] \end{cases}$$

Where should A cut the cake?

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Cut and Choose is not Equitable

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The Surplus Procedure

S. Brams, M. A. Jones and C. Klamler. *Better Ways to Cut a Cake*. Notices of the AMS, 53:11, pgs. 1314-1321, 2006.

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4. Suppose a is to the left of b (Then A receives $[0, a]$ and B receives $[b, 1]$). Cut the cake a point c in $[a, b]$ at which the players receive the *same proportion* p of the cake in this interval.

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Which Cut-Point?

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Proportional equitability: $c = \frac{7}{16}$

Equitability: $e = \frac{3}{7}$

Surplus Procedure

A procedure is **strategy-proof** if maximin players always have an incentive to let $f_A = v_A$ and $f_B = v_B$.

Theorem. The Surplus Procedure (with the proportional equitability cut-point c) is strategy-proof, whereas any procedure that makes e the cut-point is strategy-vulnerable.

Fair Division and Districting

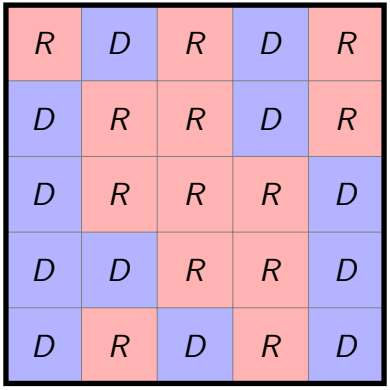
Gerrymandering

The term ‘gerrymander’ (gerry-mander) is a portmanteau dates back to early 19th century Massachusetts. The governor at the time, Elbridge Gerry, along with the State Senate, passed redistricting plans that heavily favored the Democratic-Republicans (opposing the Federalist Party) in elections, and featured particularly strange district shapes.

Metric Geometry and Gerrymandering Group

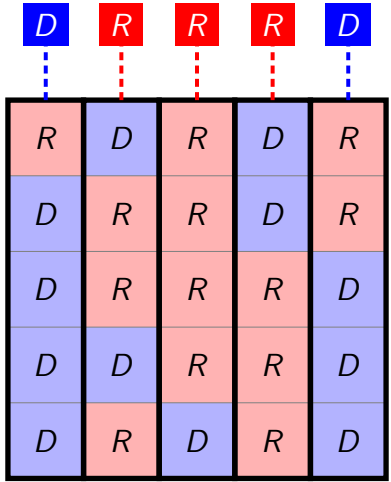
https://mggg.org/

The Problem



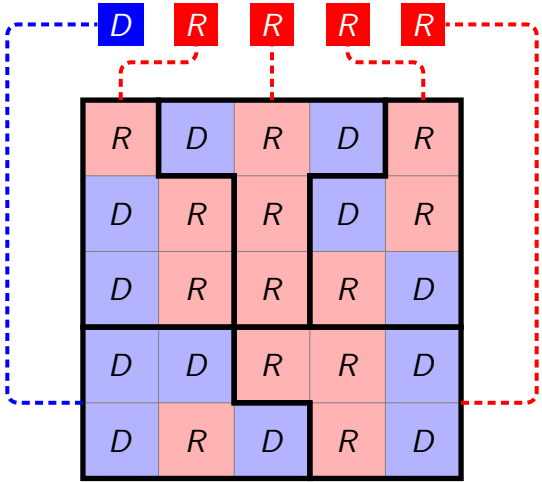
13/25 of the population will vote for R and 12/25 of the voters will vote for D.

The Problem



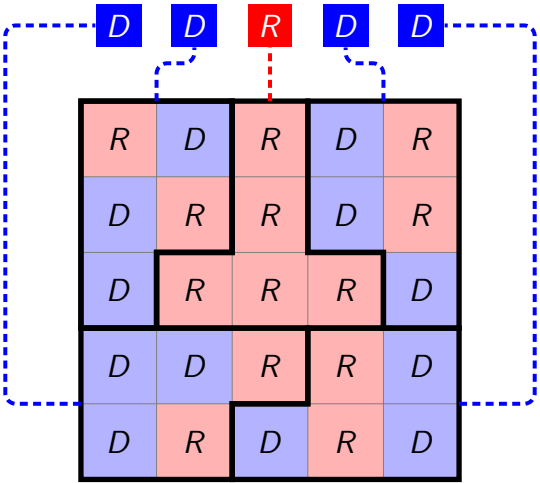
R wins 3 out of the 5 districts.

The Problem



R wins 4 out of the 5 districts.

The Problem



D wins 4 out of the 5 districts.

Tactics of gerrymandering

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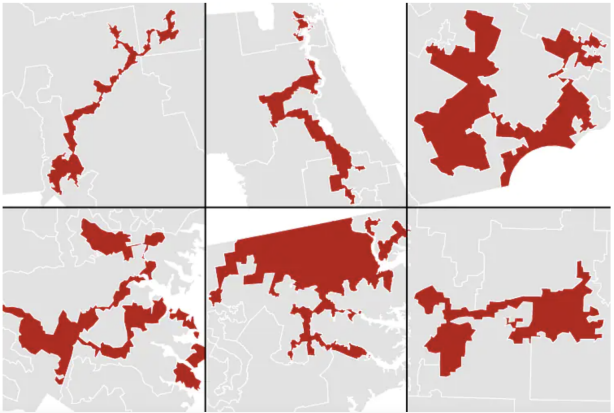
- **Packing:** taking a group/community that is spread over a wider area and concentrating them into one (or a few) districts. This minimizing how many electoral races the group can influence.

The Washington Post
Democracy Dies in Darkness

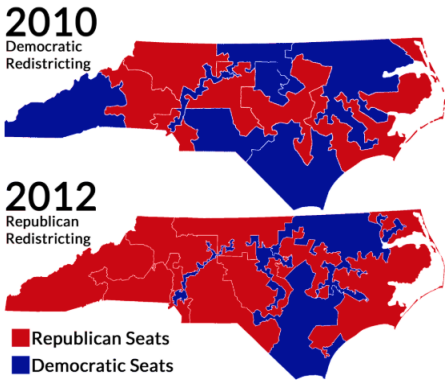
America's most gerrymandered congressional districts

By Christopher Ingraham

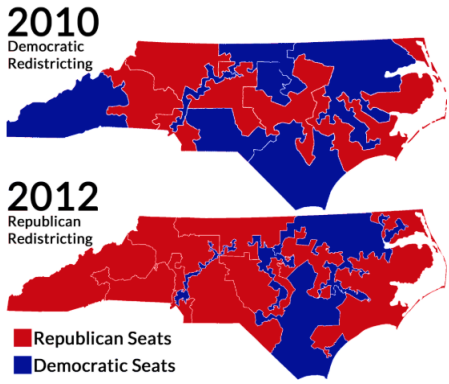
May 15, 2014 at 10:22 a.m. EDT



North Carolina

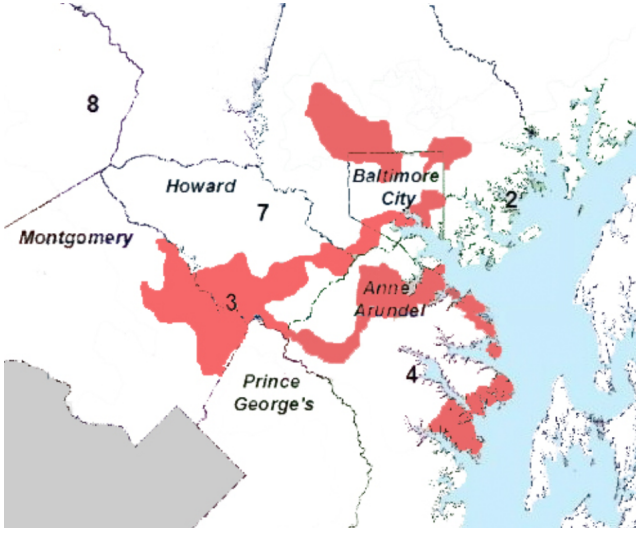


North Carolina

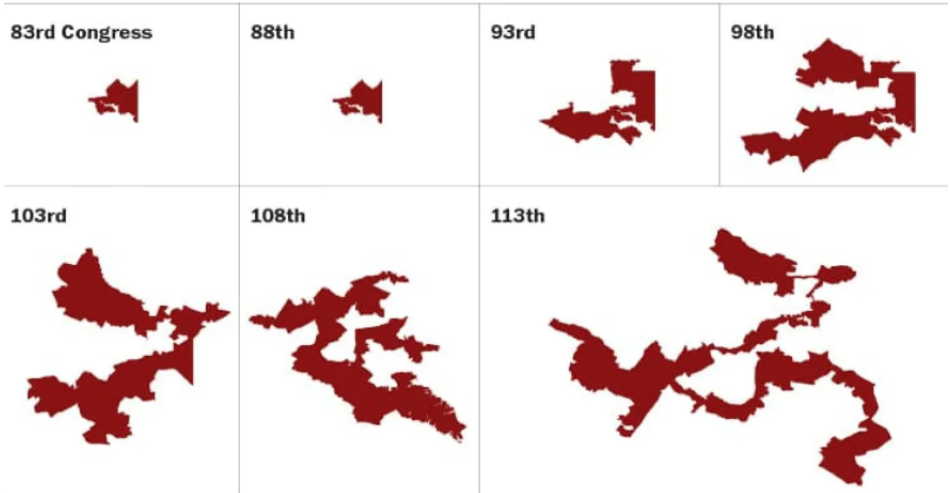


"[I] proposed that [the Commitee] draw the maps to give a partisan advantage to 10 Republicans and 3 Democrats because [I] did not believe it [would be] possible to draw a map with 11 Republicans and 2 Democrats" -David Lewis, co-chair of the NCGA's Joint Select Committee on Congressional Redistricting

Maryland 3rd



THE EVOLUTION OF MARYLAND'S THIRD DISTRICT



Kinds of Gerrymandering

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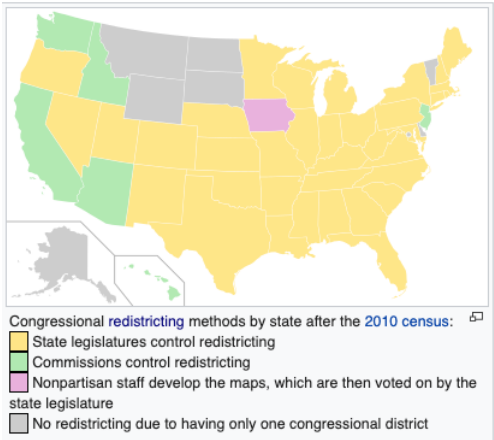
- **Partisan Gerrymandering:** redistricting that favors one political party or disfavors another. What is commonly referred to by 'gerrymandering.' Examples date back to elections for the First US Congress!
- **Bipartisan (Incumbent) Gerrymandering:** drawing politically safe districts that favor incumbents regardless of political affiliation (i.e., packing). Example: 2000's California. Not very common overall.

Non-Partisan Commission



"Ideally, of course, it would be preferable to eliminate gerrymandering entirely by having an independent commission draw the district lines of a state..." (Brams, p. 69)

S. Brams (2020). *Making Partisan Gerrymandering Fair: One Old and Two New Methods*. Social Science Quarterly, 101(1), pp. 68 - 72.

Brams's solution: Divide the state into two parts—each roughly proportional in population to the number of votes the Democratic and Republican candidates received in the last congressional elections—and allow each party to gerrymander its part unilaterally.

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For example, if a state has 10 congressional districts and the Democratic Party candidates won 60 percent of the statewide vote in the previous congressional elections, it would receive a part that comprises 60 percent of the population (and six seats) and the Republican Party a part that contains 40 percent of the population (and four seats).

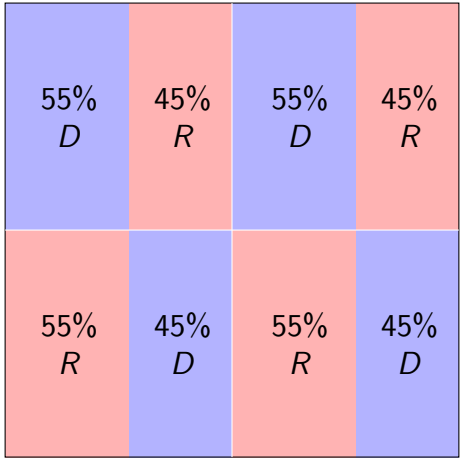
Dividing the State

1. Draw a line through the state creating a partisan replica of the entire state, insofar as possible, in each of the two parts. In the preceding example, each part would be 60 percent Democratic and 40 percent Republican, duplicating the partisan makeup of the entire state.

2. "Divide-and-choose": Give the majority party the right to proportionally divide the state into the two parts. The minority party would then choose whether its part is clockwise or counterclockwise of the radius of a circle that encompasses the state and be able to gerrymander this part unilaterally.

For this purpose, embed a state in a circle whose center is the population center of mass of the state. This is the physical point in a state about which the population is evenly distributed.

Put another way, this center equalizes the "pull" of the population in all directions. It may be in a big city, or it may be between two or more cities in an underpopulated part of the state. It is the point at which the entire population of the state can be concentrated that balances its pull in all directions.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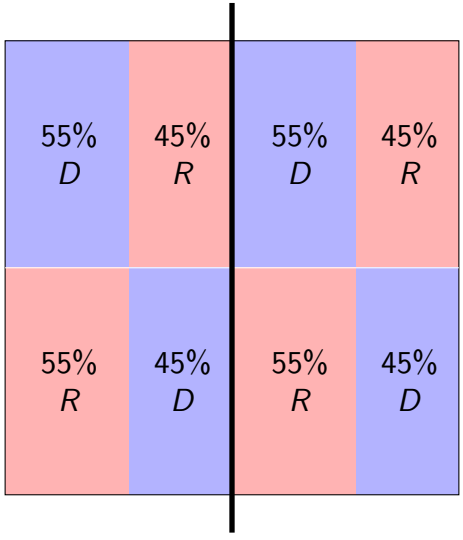
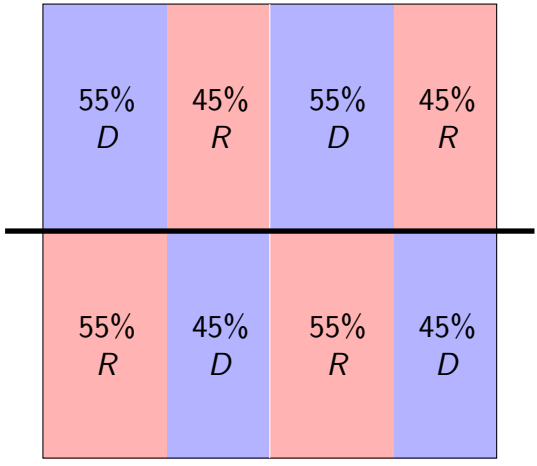
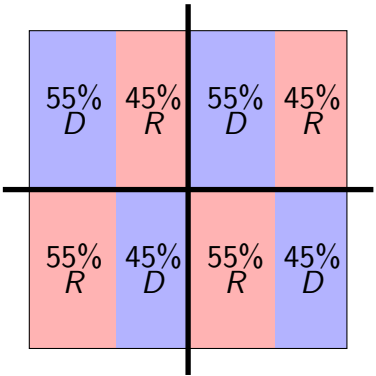


50%, 50%



55%, 45%



If the parties are risk averse, it seems likely that they would choose the horizontal division, ensuring each of one district.

If they are risk prone, they would choose the vertical division, giving each a *chance* of winning both districts.

Thus, the two methods may lead to very different outcomes, with the first giving the parties no leeway in choosing a division and the second allowing them some choice.

Neither method is a panacea in making gerrymandering fair by ensuring that each party can obtain a number of districts approximately proportional to its vote share in the last congressional elections. But each should facilitate the parties' ability to prevent extreme gerrymanders, whereby the party in control of a state is able to gerrymander the entire state and thereby win a disproportionate number of districts.