PHPE 308M/PHIL 209F
Fairness

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Fair Representation Act

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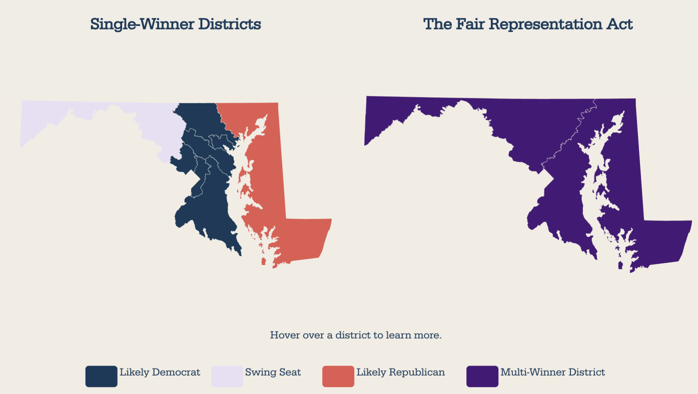
The bill requires (1) that ranked choice voting be used for all elections for Senators and Members of the House of Representatives, (2) that states entitled to six or more Representatives establish districts such that three to five Representatives are elected from each district, and (3) that states entitled to fewer than six Representatives elect all Representatives on an at-large basis.

https://fairvote.org/our-reforms/fair-representation-act/

Fair Representation Act in Maryland



https://fairvote.org/the-fair-representation-act-in-maryland/

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Is this a good idea?

Multiwinner Voting

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- An online retailer needs to pick k out of m products to display on the company's front page, given (likely) customer preferences.
- In a national election, k out of m candidates running need to be chosen to form the new parliament, based on voter preferences.

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3. **Proportional representation**. Try to find a committee of say, k representatives, each associated with a constituency of roughly $1/k$ the size of the total voting population. If two candidates are clones of each other and one is on the committee, then whether the other should be on the committee depends on whether this is necessary for proportionality.

Multiwinner Ballots

1. Approval ballots: Voters submit a *set* of candidates.
2. Ranked ballots: Voters submit a *ranking* of the candidates.

Ranked Ballots

v_1	v_2	v_3
a	b	c
c	c	b
b	a	a

- Voter v_1 ranks a above c above b
- Voter v_2 ranks b above c above a
- Voter v_3 ranks c above b above a

Ranked Ballots

3	2	1
a	b	c
c	c	b
b	a	a

- 3 voters rank a above c above b
- 2 voters rank b above c above a
- 1 voter ranks c above b above a

Approval Ballots

v_1: $\{a, c\}$

v_2: $\{b\}$

v_3: $\{a, b, c\}$

- v_1 approves of a and c
- v_2 approves of b
- v_3 approves of a, b, and c

Examples based on Plurality

- **Single Nontransferable Vote** (SNTV) or k-**plurality**: elect those k-element committees obtained by adding to the committee all those with highest plurality score, then all those with second highest plurality score, etc.

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- **Sequential Plurality**: elect those k-element committees obtained by adding to the committee the plurality winners, then removing those winners from the profile, then adding to the committee the plurality winners from the reduced profile, etc.

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Suppose we want to use the plurality rule to elect $k = 2$ winners:

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- Sequential Plurality: a wins first round, then c. So $\{a, c\}$ wins.

Bloc Voting

To elect a committee of size k: Each voter assigns a number of points to a committee equal to the number of candidates in the committee in the top k positions of the voters' rankings. The committees with the most points win.

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According to Bloc voting, for a committee of size 2, both $\{a, c\}$ and $\{b, c\}$ are tied for the win.

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Problem: Suppose that $k = 3$ and that:

- 51% approve $\{a, b, c\}$ and
- 49% approve $\{d\}$.

Then AV elects $\{a, b, c\}$, even though the 49% 'deserve' d.

Proportionality

Proportional Approval Voting (PAV). Each voter's approval ballot A assigns points to a committee C based on how many approved candidates are elected:

- 1 approved candidate in C: 1 point
- 2 approved candidates in C: $1 + \frac{1}{2}$ points
- 3 approved candidates in C: $1 + \frac{1}{2} + \frac{1}{3}$ points
- and so on.

In general, the ballot A assigns C the following points: $\sum_{i=1}^{|A \cap C|} \frac{1}{i}$. The committee(s) with the most points win.

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  (Similarly for $\{a, c, d\}$ and $\{b, c, d\}$).

So, $\{a, b, d\}$, $\{a, c, d\}$ and $\{b, c, d\}$ are tied for the PAV winners.

Single Transferrable Vote (STV)

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STV is not a specific voting method, but rather a family of voting methods that share certain principles:

1. Every voter should be allowed to allocate all of his vote to the candidate of his choice.
2. If a candidate has more than enough votes to be elected, then surplus votes should be transferred to the next available candidates in the rankings of those who voted for the candidate with the surplus.
3. If the candidate to whom a vote is presently allocated is excluded, then that vote should be transferred to the next available candidate in that voter's ranking.

Nicolaus Tideman and Daniel Richardson (2000). *Better voting methods through technology: The refinement-manageability trade-off in the single transferable vote*. Public Choice, 103, pp. 13 - 34.

Single Transferrable Vote (STV)

STV is a multistage elimination rule that works as follows. Suppose that there are n candidates and k candidates to elect. If there is a candidate c whose Plurality score is at least $q = \lfloor \frac{n}{k+1} \rfloor + 1$ (the so-called Droop quota), we do the following:

(a) include c in the winning committee,

(b) delete q votes where c is ranked first, and

(c) remove c from all the remaining votes.

If each candidate's Plurality score is less than q, a candidate with the lowest Plurality score is deleted from all votes.

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The different STV methods vary primarily in how much of which surplus votes are transferred and in the meanings that are attached to "enough votes to be elected" and "the next available candidate".

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- When just one candidate is elected, a candidate needs barely more than half the votes to be assured election.
- If two candidates are to be elected, then any candidate with more than a third of the votes ought to be assured election on the basis that there can be at most one other candidate (who can be given the other position) who will receive as many votes.

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- If two candidates are to be elected, then any candidate with more than a third of the votes ought to be assured election on the basis that there can be at most one other candidate (who can be given the other position) who will receive as many votes.
- In general, if there are k positions to be filled, then there can be at most k candidates who have more than $n/(k+1)$ votes.

Hare Quota

The **Hare quota** is n/k where n is the number of voters and k is the number of seats to be filled.

Example: STV (Hare Quota)

Suppose that there are 48 voters and $k = 3$. Using the Hare quota, a candidate needs at least $q = 48/3 = 16$ votes to be elected.

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c	d	a	b
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Then, a will be elected, *but none of the votes will be transferred to* b *since there is no surplus*. b will be removed, and c and d will be elected.

So the candidates elected will be $\{a, c, d\}$.

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d	c	b	a

Then, a will be elected, *a surplus of three votes will be transferred to* b. So, b will be elected. One of the remaining two candidates will be selected at random.

So the winning committees are $\{a, b, c\}$ and $\{a, b, d\}$.